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Deepening Preservice Elementary Teachers' Mathematical Knowledge for Teaching Through Writing and Computer-Mediated Interinstitutional Conversations

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In this practitioner-focused article, the authors present a mathematical writing activity for preservice elementary school teachers. The products of this activity, referred to as "journals," are detailed, representation-rich explanations of problems solved in small groups. These journals were originally exchanged across institutions in a traditional peer review process; and the addition of a computer-mediated communication platform transformed the peer reviews into interinstitutional conversations. Analysis of the journals and the conversations they sparked brought to light four opportunities to develop mathematical knowledge for teaching. Evidence in the form of journal and review excerpts suggests that the process, enhanced by the computer-mediated communication, enabled preservice teachers to deepen their understanding of foundational concepts from the elementary school curriculum, to communicate mathematics more effectively to others, to better make sense of the mathematical thinking and writing of others, and to incorporate their colleagues' suggestions into their future writing. Suggestions and considerations for implementing this activity are discussed.

Mathematical communication is a crucial practice for supporting mathematical understanding (Hiebert et al., 1997; Martin & Polly, 2016; Pugalee, 2001). Furthermore, it is an essential skill for teachers (Association of Mathematics Teacher Educators [AMTE], 2017). As mathematics teacher educators, we have been exploring strategies that utilize oral and written communication to develop elementary preservice teachers' (PSTs') understanding of mathematics and their ability to communicate it well. This practitioner-focused article showcases a collection of mathematical activities that we have come to call the APEX Cycle. We describe our examination of opportunities it offers for PSTs to deepen their mathematical knowledge for teaching (Ball et al., 2008).

Over a period of 7 years, we refined elements of the cycle based on our analysis of PST work and feedback. One such refinement, the introduction of a computer-mediated communication platform, greatly enhanced the final phase of the cycle. The APEX Cycle culminates in an exchange of detailed written explanations of rich mathematical tasks, or "journal entries," in which PSTs review and critique the mathematical writing of others. Using online shareable documents (Google Docs) to facilitate a "digital exchange" of these journal entries transmuted the peer review process into collegial conversations where PSTs' learning continued and deepened. This article presents an innovative model for mathematics teacher educators interested in using mathematical writing to enrich their programs and to foreground the affordances of technology in doing so.

Rationale for the Innovation and Connection to the Existing Knowledge Base

The connection between mathematical communication and mathematical understanding has a long tradition in the mathematics education literature (Hiebert et al., 1997; Martin & Polly, 2016; Pugalee, 2001). For example, Hiebert et al. emphasized reflection and communication as major parts of learning mathematics with understanding. Lim and Pugalee (2004) pointed out that writing "allows students to make connections, reflect on and synthesize learning, while also engaging in authentic practices of the discipline" (p. 18).

Martin and Polly (2016) concluded that when writing was used as a tool for mathematical problem solving, the depth of student understanding was evident. Moreover, Powell et al. (2021) noted the benefits of mathematical writing as a feedback mechanism that allowed educators to "understand conceptual misconceptions and procedural mistakes and support students in organizing their thinking, connecting mathematics ideas, and developing a deeper and richer understanding of mathematics" (p. 419).

Fittingly, the standards for mathematical practices from the Common Core State Standards, the process standards from the National Council of Teachers of Mathematics (NCTM) standard documents and its *Principles to Actions* encourage mathematics teachers to support written and oral communication of all learners and doers of mathematics (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010; NCTM, 1989, 2000, 2014). The significance of communication in mathematics learning is also of international interest.

The Language and Communication in the Mathematics Classroom Topic Study Group of the Fifteenth International Congress on Mathematical Education (ICME, n.d.) explored the “relations between language, communication, or discourse with such key constructs as mathematics, learning, thinking, understanding, and reasoning” (para. 5).

One way for well-prepared beginning teachers of mathematics to demonstrate mathematical practices and processes is to communicate their ideas and understanding using mathematical language with care and precision (AMTE, 2017). The Conference Board of the Mathematical Sciences (2016) suggested the use of reading, writing, and discussion as examples of active learning strategies to enrich postsecondary mathematics education. The NCTM (2020) recommended that “[teacher] candidates organize their mathematical thinking and use the language of mathematics to express ideas precisely, both orally and in writing to multiple audiences” (p. 18). Therefore, content courses within programs for elementary preservice teachers should provide writing experiences that promote the use of language and representations appropriate for teaching children mathematics.

Writing in mathematics can help PSTs “consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas” to communicate them (NCTM, 2000, p. 61). As Hiebert et al. (1997) explained, while reflection is more of a metacognitive process where one focuses on his or her thinking, communication is a social process involving “talking, listening, writing, demonstrating, watching … participating in social interaction, sharing thoughts with others” (p. 5). Through communication, ideas become public. Individuals can “challenge each other’s ideas and ask for clarification and further explanation,” which might result in more reflection (p. 5).

Writing in mathematics may start as a reflective activity, but it often ends as communication with someone else. Therefore, concepts and reasoning need to be expressed as clearly as possible so that readers can make sense of it on their own. Thus, mathematical writing is an important activity to support PSTs as they transition from doers to teachers of mathematics. Before communicating their mathematical understandings with elementary school students, PSTs can communicate with each other. Existing platforms, such as Google Docs, present important affordances in mediating this type of early mathematical communication. Some of these affordances are described more generally in the next section.

Computer-Mediated Communication

Engaging in peer review is not a novel idea in the teacher education literature. Howard et al. (2010) argued that PSTs need to learn how to give good feedback as future teachers, so experiencing this as students is critical. Lim et al. (2021) noted that peer feedback helps PSTs to “identify their strengths and areas of growth … [and] increases collegiality and helps to shape professional practice” (p. 1). However, providing and receiving constructive, formative, honest, and critical feedback is often difficult and might require structure and support (An et al., 2009; Howard et al., 2010; Lim et al., 2021). The literature offers guidance, such as the use of computer-mediated communication, instructor facilitation, and anonymous feedback to improve PST feedback and reflection (see also Sutherland et al., 2010; Yang, 2009).

Computer-mediated communication (CMC) is “an umbrella term that encompasses various forms of human communication through networked computers, which can be synchronous or asynchronous and involve one-to-one, one-to-many, or many-to-many exchanges of text, audio, and/or video messages” (Lee & Oh, 2017, p. 1). An et al. (2009) said that “…asynchronous CMC can promote knowledge construction, critical thinking, and problem solving, through interaction and communication with fellow students and instructors” (p. 749). Howard et al. (2010) added that “asynchronous CMC often reduces fear of humiliation, allows for less stressful preparation of comments, and provides equity in participation, allowing students who may otherwise be hesitant to become active” (p. 91). By removing time constraints, asynchronous online environments can offer participants additional time to reflect not only on the feedback that they received but also on the response they will provide (Singer & Zeni, 2004; Sutherland et al., 2010).

The following sections provide information on the design, facilitation, study, and improvement of an innovative mathematical writing activity for elementary school PSTs. In particular, we discuss how CMC supported and enriched the activity.

Design of the Innovation

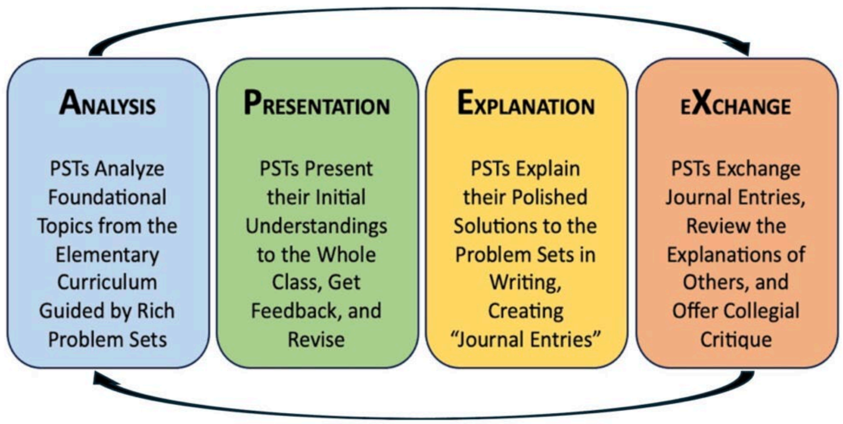
The work described here is part of a Noyce Mathematics Fellows project, which aims to increase the number of high-quality elementary and middle school mathematics teachers in a midwestern urban public school district. One of the many components of this project is the development of high-quality mathematics courses for PSTs at a university and a nearby college. The content courses are critical sites for the development of mathematical knowledge and practices that teachers use when teaching children (as recommended by Ball, 2000).

We analyzed and redesigned the Mathematics for Elementary Teachers courses at the two institutions to improve the opportunities for PSTs to develop their mathematical knowledge (Ball et al., 2008). We sought ways to incorporate mathematical communication (both oral and written) as a major part of this development. The redesign effort proceeded in an iterative fashion as the effectiveness of each new activity was analyzed and its content and facilitation were adjusted.

APEX Cycle

As the teaching in these courses shifted to a more activity-based, student-centered design, we integrated several “high-leverage practices” (Ball et al., 2009). The PSTs studied mathematics through a group of activities that we have come to call the APEX Cycle (see Figure 1 and Table 1).

Figure 1
The APEX Cycle



PSTs first engaged in an *Analysis* phase, in which they explored key mathematical concepts from the elementary school curriculum. They collaboratively investigated these mathematical ideas and shared their individual understandings. Afterwards they shifted to the *Presentation* phase. PSTs presented their group's collective understanding to the whole class. During these presentations, PSTs shared their ideas and worked to develop clear representations and language to communicate their mathematical ideas. They built on their original thinking, refining and formulating other ways to communicate it. This transitioned smoothly to the *Explanation* phase.

Each PST wrote solutions and explanations, which we call journal entries (or simply "journals"), in which they responded to the following prompts: write to a peer, include important conceptual details, coordinate any representations (pictorial or mathematical) with detailed explanations, justify your thinking, and explain not only how you worked, but the *why* behind your work (see Figure 2). Last, the PSTs *Exchanged* journals across institutions. During this exchange, PSTs reviewed a colleague's journal entry, examining it for clear and correct mathematical work and communication of mathematical ideas.

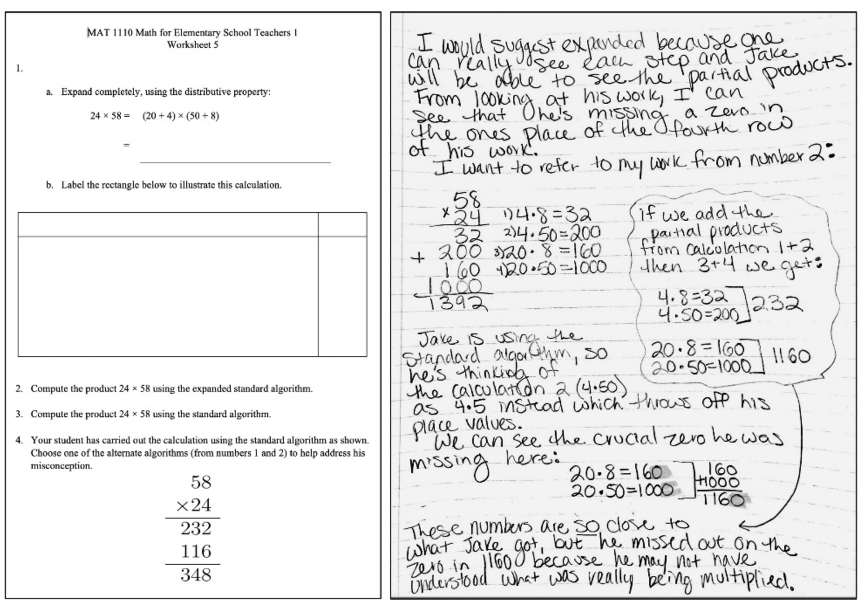
The Exchange Phase

Each term, up to two of the eight journals were exchanged so that the PSTs could make sense of their peers' mathematics, critique the written work of other PSTs for the purpose of developing skills to offer feedback in a positive and professional manner, and gain a new perspective when reflecting on their own written explanations. The initial purpose in having the PSTs exchange journals with a student from another school was to promote cross-institutional connections and to have them read the work of PSTs who were taught by another instructor. We theorized that, even though both groups of students were studying the same mathematics and using the same journal assignments, the perspectives and explanations could be different enough to require a more careful review.

Table 1
The APEX Cycle With Examples

PEX Phase	Description	Example (Whole-Number Multiplication)
Analysis	PSTs analyze foundational topics from elementary curriculum guided by rich problem sets	PSTs analyze multiplication of two-digit numbers using several approaches: Distributive Property, area model, expanded standard algorithm, standard algorithm (see Figure 2a). Also, PSTs examine student work to identify the error and make recommendations to repair the error. Analysis is done individually then with peers
Presentation	PSTs present initial understandings to the whole class, get feedback, and revise	PSTs present their work to the class clarifying each approach to two-digit multiplication, representations and explanations, making clear the similarities and differences among the approaches, recommending ways to explain and repair student error
Explanation	PSTs explain their polished solutions to the problem sets in writing, creating "journal entries"	PSTs write improved solutions to each approach to two-digit multiplication and student error including representations and explanations fine-tuned during the presentation phase (see Figure 2b)
Exchange	PSTs exchange journal entries, review the explanations of others, and offer collegial critique	PSTs review and make recommendations for improvement. Figure 3 demonstrates a question posed to a peer asking more explanation about the steps taken in two-digit multiplication

Figure 2
Sample Worksheet and Page From a Journal Entry



The exchange offered the PSTs the opportunity to review others' journals, to make sense of and offer critique of the mathematical explanations and representations, and to make suggestions for improvement. To support this work, the instructors offered five prompts to guide the PSTs' review. The first four prompts were used for both physical and digital exchanges (described in the next section). The final two prompts were specific to the physical exchanges. The final version of the exchange prompts is included in the appendix.

- What part(s) of this journal do you feel is (are) done well? Please be as specific as possible.
- What part(s) of this journal do you feel could be improved? Please offer specific suggestions for improvement.
- What are the important mathematical ideas that you see used?
- Are these ideas used correctly? Are there ideas that you feel deserve more explanation? If so, what would you suggest adding?
- If you could ask the author one question about this work, what would it be?
- The space below is offered as a place for you to write a friendly note to the author.

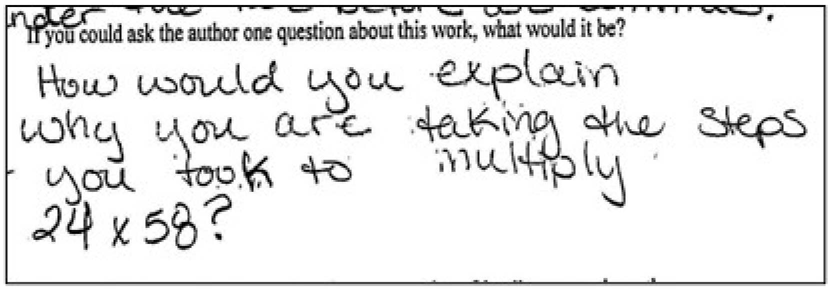
Although the PSTs found this exchange and review process interesting, the logistics of copying and transporting physical copies of the journals and reviews back and forth limited the timeliness and effectiveness of the experience. Most importantly, the physical exchange offered no opportunity for the PSTs to respond to the comments, to reply to questions posed, or to interactwith their partner in any additional way.

Figure 3 shows a question posed by the reviewer, "How would you explain why you are taking the steps that you took to multiply 24 x 58?" This is a reasonable question, one for which she would not receive an answer. Thus, after 2 years of exchanging journals physically, we began to search for a more practical and productive method of exchanging the PSTs' written work. Drawing on the computer-mediated communication literature, we piloted a digitalexchange of journals.

Digital Exchanges

In our search for an effective way to facilitate a digital exchange, we reviewed several CMC options. Google Docs eliminated the logistical and clerical challenges of exchanging physical journals and it provided an easy-to-navigate system for the PSTs to read and comment on their partner's journals.

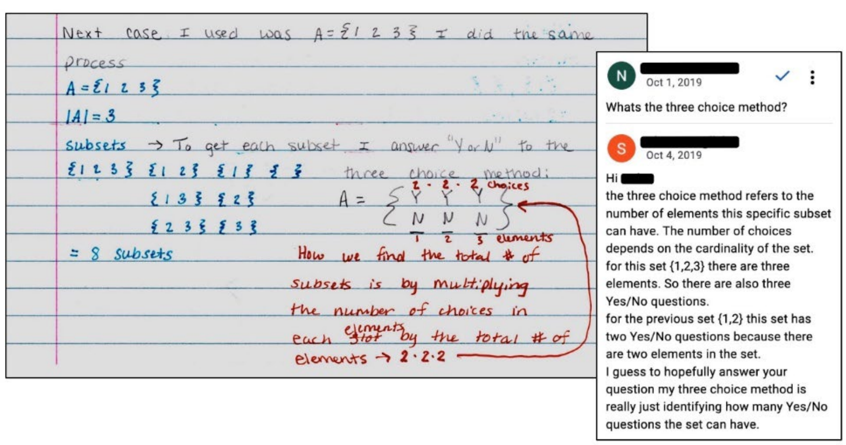
Figure 3
Physical Exchange Review Form With Question to Author



Initially, we asked the PSTs to review their partner's journal using prompts similar to the physical exchange, and many PSTs went beyond our directions. Some even exchanged contact information so that they could communicate more about their work. This prompted us to add an additional layer to the review: we asked the PSTs to respond to their review partner's comments within the Google Doc. This approach often provided opportunities for the PSTs to rethink and clarify their mathematical explanations.

For example, in one APEX cycle from the fall 2019 semester, the PSTs experimented with finite sets to determine if there was a pattern for the number of subsets that could be formed from a given set. In his journal entry for this problem, Scott (names are pseudonyms throughout and first initials appear in the Google Doc comments) justified his claim that a set with three elements has eight subsets by appealing to what he called the "three choice method" (see Figure 4).

Figure 4
Author Responding to Reviewer Question during a Digital Exchange



In her review of Scott's journal entry, Nahla sought clarification by asking, "Whats [sic] the three choice method?" Scott, now aware that he used terminology and an approach that were unfamiliar to Nahla, responded with additional details. He added that "the number of choices depends on the cardinality of the set. For this set {1, 2, 3}, there are three elements. So there are also three Yes/No questions."

Scott's second attempt to better explain his use of the fundamental counting principle shows the potential for authentic mathematical dialog afforded by the digital exchange. We realized that these digital conversations also represented an additional opportunity to deepen PSTs' mathematical knowledge; thus, we began a more in-depth analysis of these interactions.

Evidence of Impact of Innovation

A brief description of our method of documentation and analysis of the PSTs' computer-mediated communication is offered here, so that teacher educators who wish to implement this type of activity might better recognize the impact on their own students' learning. Our main goal is to identify ways to facilitate PSTs' learning and communication of mathematics appropriate for teaching. After implementing the digital exchange, we analyzed the PSTs' interactions that it supported.

We gathered evidence from the Mathematics for Elementary Teachers courses taught more than 28 times, collectively, at both institutions over a 7-year period. Students typically engaged in eight APEX cycles per semester, approximately 224 cycles in total. We coded independently, using a constant comparative approach (Merriam, 1988), the 2,541 comments from the 378 journals that were exchanged digitally between 2018 and 2022 (see Table 2). While conducting this analysis, our aim was to improve the APEX cycle, especially the "X" phase, and our time was funded as part of multiple projects. We acknowledge that this level of analysis might not be reasonable for mathematics teacher educators (MTEs) as part of their daily work and reflection. When looking for evidence for this kind of innovation, MTEs could elicit PSTs' reflections via surveys or essay-type exam questions, compare their PSTs' journal and exam explanations of mathematical concepts, or observe PSTs' in-class mathematical communication.

Differences in individual codings were discussed and reconciled. While some codes that surfaced were expected, for example *Explanation* and *Representation*, we identified other codes that were grounded in the data such as *Future Teaching* (applied when PSTs referred to their future teaching practice or the teaching of children), and *Social* (applied when PSTs used social pleasantries to engage with their partners or mitigate critical feedback).

Table 2
Initial Coding of the Digital Exchanges

Semester-Exchange	Journal Entries	Comments	Average # of Comments	Explanation (Critical)	Explanation	Explanation (Positive)	Dialogue & Interaction	Probing Questions	Math-Specific	Representation (Critical)	Representation	Representation (Positive)	Step-By-Step Process	Color	Layout	Helped My Understanding	Another Way of Thinking	Understand Your Thought Process	Future Teaching	Social
Fall 2018-1	39	224	5.7	48	0	66	21	38	43	21	4	44	24	7	17	12	15	13	7	13
Fall 2018-2	25	137	5.5	23	0	40	56	7	53	4	0	31	43	21	19	13	8	6	24	4
Fall 2018-3	15	100	6.7	11	0	31	9	11	38	13	1	30	6	2	3	3	4	3	15	4
Winter 2019-1	28	198	7.1	45	21	58	64	35	47	14	8	42	12	12	11	11	16	1	11	65
Winter 2019-2	30	285	9.5	61	29	126	127	59	178	7	9	69	22	8	19	28	40	7	14	59
Fall 2019-1	28	184	6.6	25	1	63	63	21	46	11	4	25	13	8	25	33	2	7	7	19
Fall 2019-2	27	151	5.6	23	1	47	47	16	61	11	0	33	5	8	12	9	10	0	14	10
Winter 2020	25	139	5.6	23	15	52	52	6	49	10	13	24	30	15	25	13	7	2	7	53
Fall 2021	39	239	6.1	75	7	137	137	10	106	16	6	26	44	24	20	33	32	11	68	98
Winter 2022-1	45	406	9.0	73	17	184	184	24	156	32	21	78	38	38	28	38	53	13	62	223
Winter 2022-2	41	478	11.7	130	15	128	128	116	294	78	16	90	16	11	13	11	38	3	32	241
Totals	342	2541	7.4	537	106	932	932	343	1071	217	82	492	253	154	192	204	225	63	261	789

Our analysis led to organizing the codes into emergent themes and analytic categories, which we identified as Opportunities to Learn (OTLs). The analysis concluded by making conjectures regarding affordances of CMC that supported these OTLs. Through our analysis, we discovered that the writing and exchange of the journals enabled PSTs to build their mathematical knowledge in multiple ways. Writing, exchanging, and engaging in conversations about journals proffer four OTLs (Nazelli et al., 2024):

1. Writing a mathematical journal to deepen one's understanding,
2. Writing for someone else to communicate one's thinking,
3. Reading someone else's journals, making sense of others' thinking, and providing comments, and
4. Improving one's future mathematical writing.

For example, in one Google Doc comment interaction, the reviewer stated, "Drawing out each part of the rectangle and coloring them separately makes it easier to understand. Especially as a visual learner, this helped me." The author replied, "When I was solving this problem, I did not organize it in this way, but I found it helpful for my journal explanation." We applied the codes *Representation (Positive)*, *Color*, *Helped my Understanding*, and *Another Way of Thinking*. We found that the dialog reflected an opportunity to learn by writing for someone else. The author shifted her approach to better explain the mathematics. The effectiveness of this different representation was validated by the reviewer.

Table 3 shows two other coding progressions and their connections to OTLs. Having developed and demonstrated the connections between our initial codes and OTLs in Tables 2 and 3, we focus on the OTLs afforded by the writing and exchange by CMC.

Table 3
Sample Coding Progression of PST Comments and Connection to Opportunities to Learn

PST Comment	Initial Codes	Opportunity to Learn
"I really liked reading through your journal because it allowed me to reflect and see a different way of explaining."	Explanation (Positive) Another Way of Thinking	Making Sense of Others' Thinking
"[Your] journal... helped me see how you can use base 10 blocks to break down both the multiplication and division problems, and I plan on using your methods when making changes to my journal."	Representation (Positive) Helped my Understanding	Deepening One's Understanding Improving One's Future Mathematical Writing

CMC and OTLs

Completing the journal reviews in the digital environment allowed PSTs to (a) interact with someone else's journal entry to make sense of others' thinking (OTL 3), (b) formulate questioning and feedback to improve each other's mathematical knowledge (OTL 3), and (c) have a dialogabout each other's understanding (OTL 2). These all build on the PSTs' writing about mathematics now (OTL 1) and in the future (OTL 4). A journal exchange and review between two PSTs, Mona and Lola, is presented here to exemplify how their interactions illustrated these OTLs. Their discussions are representative of the larger collection of interinstitutional conversations and make visible multiple OTLs.

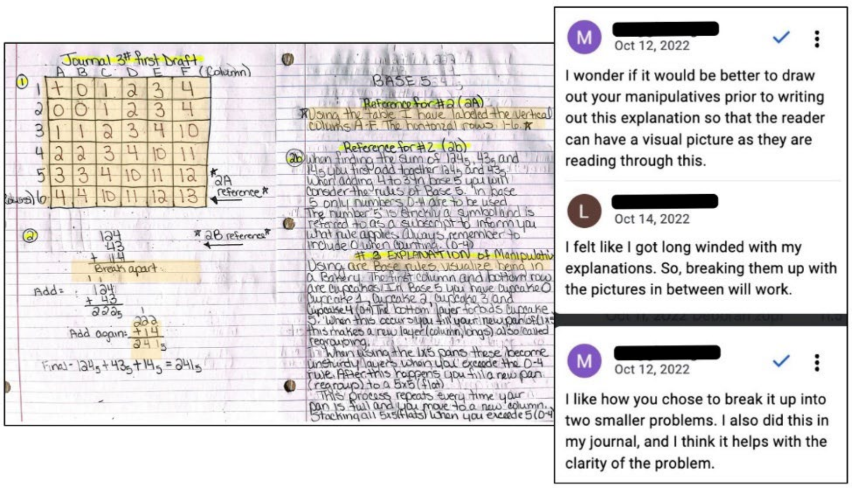
The Case of Mona and Lola

Lola began the semester noting that she had never been successful in mathematics. Writing about mathematics seemed to help her engage with the material and make sense of the concepts in the course. Mona, in contrast, was confident in her abilities to do mathematics, and writing allowed her to delve deeper into why things work.

The content of their journal exchange involved addition of base-5 numerals. Lola wrote extensive explanations, and she created scenarios to help her make sense of base-5 addition. Her journal shows how this write-up deepened her understanding (OTL 1). Mona commented on Lola's explanations and made a recommendation for her to include drawings to support her mathematical thinking (see Figure 5). Mona wrote, “I wonder if it would be better to draw out your manipulatives prior to writing out this explanation so that the reader can have a visual picture as they are reading through this.”

Their conversation appears to acknowledge that the work is correct, but Mona encouraged Lola to add more detail to help the reader understand base-5 addition. In this exchange, Mona pushed Lola to make her thinking more visible to the reader (OTL 2). Mona’s final comment, indicating that both she and Lola worked in a similar way, acknowledged Lola’s work, an important component for her developing confidence about her understanding and writing.

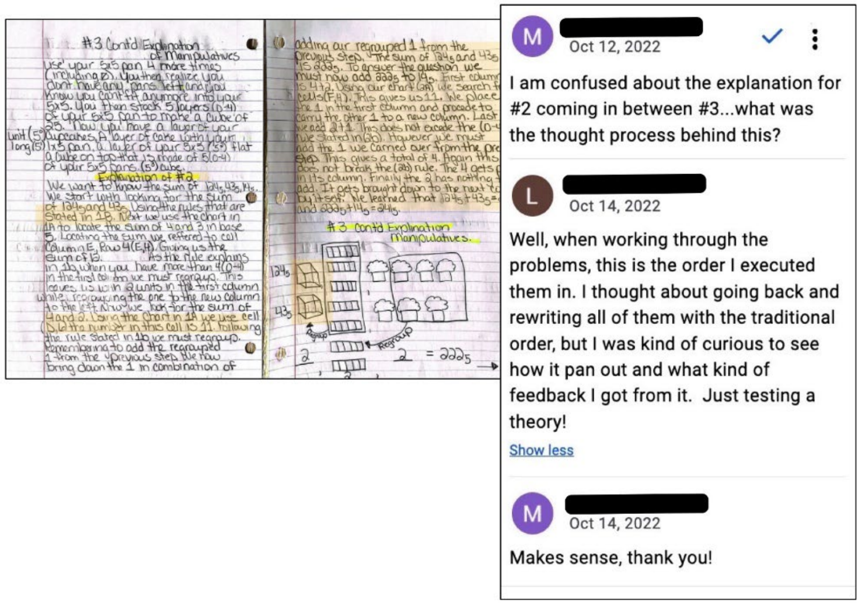
Figure 5
Lola and Mona Discuss Lola’s Work via Google Docs



The conversation continued with Mona questioning the layout of Lola’s work on the second and third problems. Lola stated that she arranged her work in the order in which she completed it. It seems that Lola backed into her understanding for Problem 2 by completing Problem 3 first (note Mona’s confusion and Lola’s response in Figure 6). Lola stated that she did, in fact, begin with Problem 3 and worked Problem 2 using what she had learned from Problem 3. She intentionally did not rewrite her work to “[test] a theory.” We do not know more about Lola’s thinking. However, we do know that the layout was intentional in the way that it supported and enhanced her mathematical thinking (OTL 1).

Mona continued to think through Lola’s work on the fourth problem from the same activity, which asked the PSTs to add $124_5 + 43_5 + 14_5$ using expanded form. Here, Lola wrote, “124 base 5 = 1(25) + 2(5) + 4(1).” Mona noted that Lola’s “expanded forms are not totally written in base-5” (see Figure 7). Mona and Lola were thinking differently about the expanded notation for 124 base-5. Lola noted her confusion, rewriting her expanded form including base-5 block language to support her work. She wrote, “expanded form for 124 base 5 is 1(25) “on[e] flat” + 2(5) “two longs” + 4(1) “four units” right?”

Figure 6
Mona and Lola Discuss the Layout of Problems 2 and 3



Mona responded to Lola's clarification:

> If you want to write the expanded for[m] *completely in base 5* [emphasis added] then you wouldn't use the number '5' anywhere because that is written 10_5. This is also why I chose to write 25 as 100_5 … just keeping everything in base-5.

The use of Google Docs allowed Lola to offer why she used "25" and "5," clarifying with base-5 block representations *and* Mona's further clarification for her use of 10 base-5 and 100 base-5 for base-5 place values (see Figure 7). This exchange highlighted how three of the OTLs were woven together in this excerpt of their conversation. Lola provided an in-depth explanation for her work (OTL 1) with the goal of revealing her mathematical thinking (OTL 2), which was not clear to Mona (OTL 3).

When reviewing Mona's journal, Lola said that she was confused about the expanded form. The conversations in both Lola's and Mona's journals demonstrated the PSTs reading someone else's work, making sense of other's thinking, and responding (OTL 3). This work appeared to have helped Lola understand the work with expanded notation as well as developed Mona's ability to read Lola's work, make sense of it, and comment to improve Lola's thinking (OTLs 1, 2, & 3) (see Figure 8). Further, evidence suggests that both Lola and Mona used their partner's comments to improve their writing. When Lola confessed that she was confused about the expanded form, Mona noted that she "will make sure [she is] clarifying terms in the future." This interaction allowed Mona to see the importance of being clear about language and representations so that the reader could read and comprehend the written mathematics. Awareness of this important aspect opened the opportunity for Mona to improve her future writing (OTL 4).

Figure 7
Lola and Mona's Discussion of the Expanded Form for 124 base-5

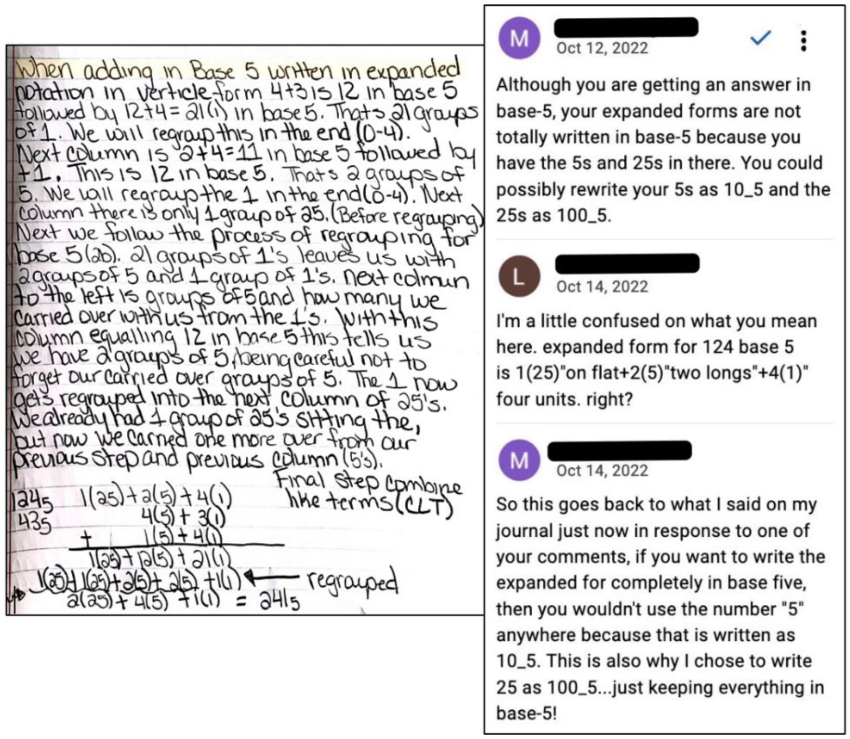
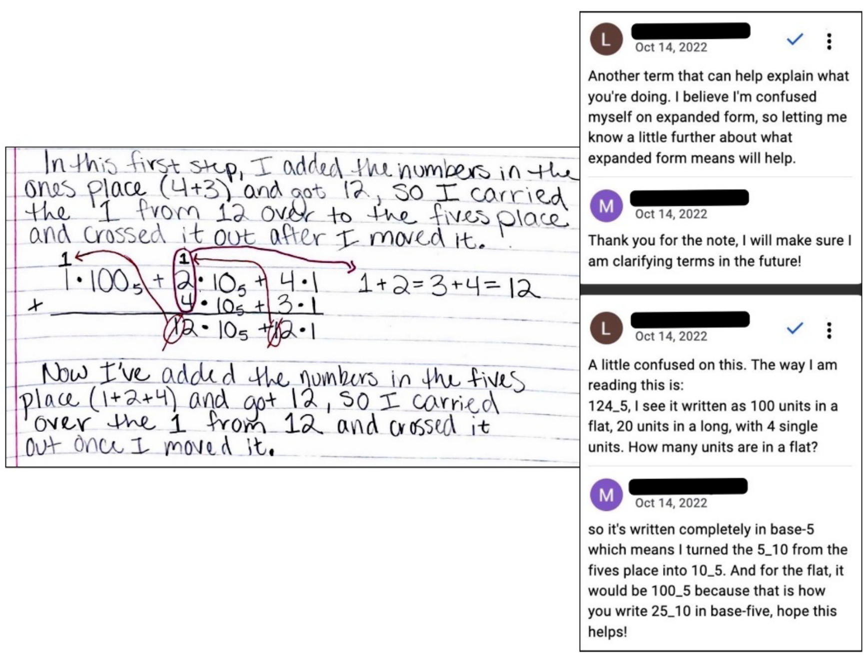
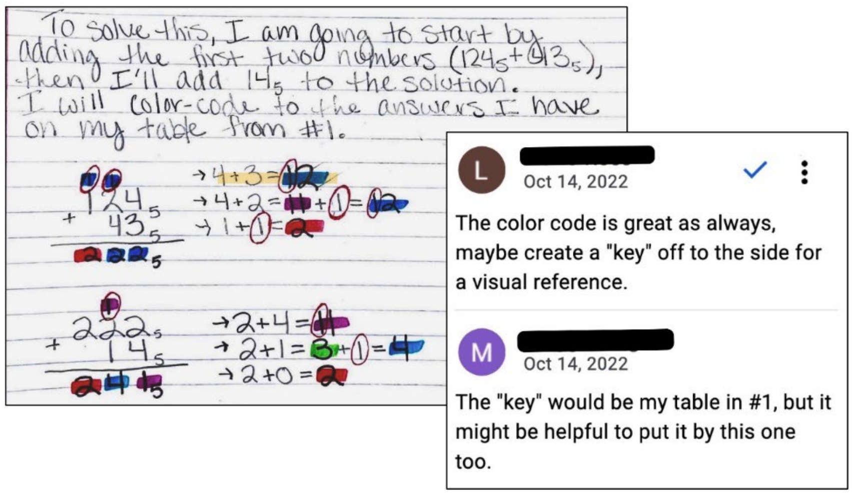


Figure 8
Lola Questions Mona's Use of Expanded Form When Adding 124_5 and 43_5



Lola's reading of Mona's journal provoked many questions that seem to indicate the fragile nature of Lola's understanding, her need for detailed representations and explanations, and her reliance on terminology consistent with her own. Figure 9 shows that Lola suggested a key to the colors used in the work. Mona noted that she used the same color coding as in Problem 1 and added that repeating the key might be helpful to readers. This addition would be an improvement to her writing (OTL 4) as she stated, "I will make sure I am clarifying terms in the future."

Figure 9
Lola Requests a Key to Make Sense of the Color Coding



In the interaction seen in Figure 10, Lola asked Mona to explain "the carrying process … what it is, and why you are carrying it." She went further by stating that 2-digit numbers were not allowed in base-5. This was a misconception that several of Lola's classmates shared. Mona agreed that she might have explained regrouping and corrected Lola's error regarding using 2-digit numbers in base-5. Mona's example was concise and directly addressed the misconception. This was an opportunity to learn for Lola, as she was able to reconsider the misconception via a discussion with a peer (OTL 3).

Mona and Lola completed their conversation by noting their use of manipulatives and Lola's creation of a bakery scene (see Figure 11). Lola shared her goal of teaching kindergarten and how this impacted her writing. Throughout Lola and Mona's work, there was conversation about parts of the work that challenged them, as well as language and procedures that needed to be made clearer (OTL 3).

Analysis of Mona's and Lola's exchange revealed how their interactions provided many opportunities to learn. As we studied this case and the other exchanges over several years, we identified six ways the digital format enhanced the interactions among PSTs.

Figure 10
Lola's Review Makes Visible Her Confusion of Base-5 Numerals

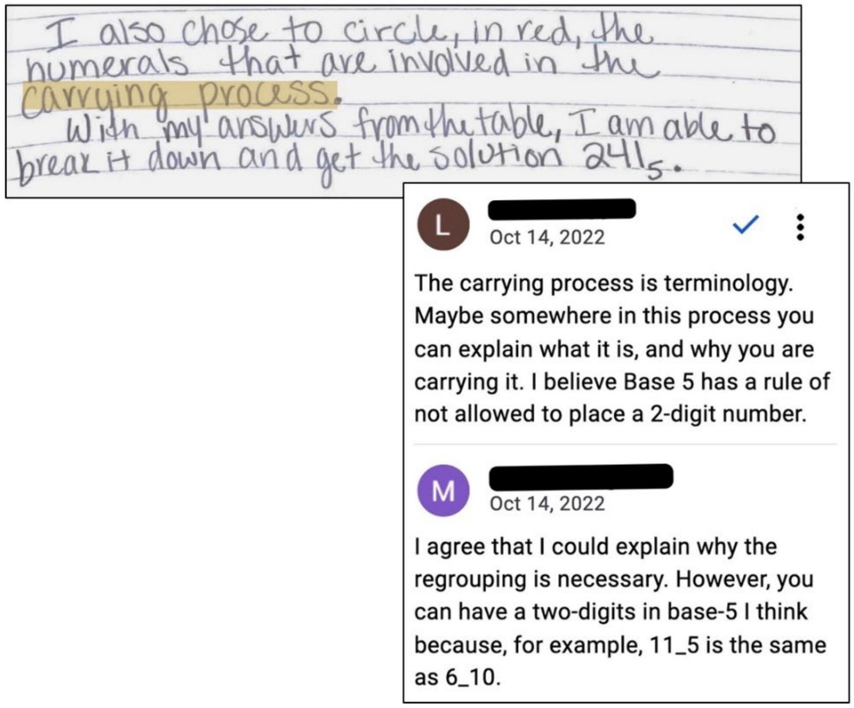
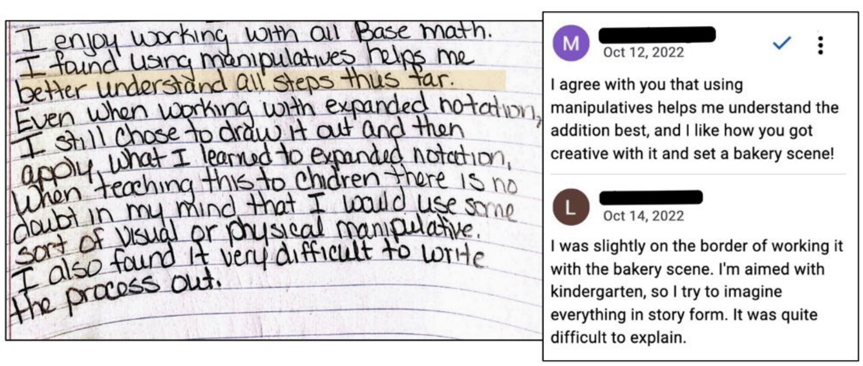


Figure 11
Lola and Mona Share Final Thoughts



Discussion of the Impact

Standard P.1 in the AMTE's (2017) *Standards for Preparing Teachers of Mathematics* stated, "An effective teacher preparation program benefits from an interdisciplinary collaborative partnership that is a shared endeavor focused on the preparation of mathematics teachers who are well prepared to improve Pre-K–12 student learning in mathematics from multiple perspectives" (p. 27). With this standard in mind, we see CMC as a great asset to foster such collaboration. The following list technical affordances of the digital platform is followed by elaborate each affordance. The digital platform enhanced the mathematical communication across institutions by allowing PSTs to:

1. see each other's work in color instead of black and white as in physical exchanges,
2. comment on specific parts of the journal by highlighting it,
3. see the mathematical work and comments together in one unified document,
4. decide the appropriate length of comments instead of being limited in the physical exchange,
5. take their time before commenting/responding, and
6. have a conversation with back-and-forths in a timely manner.

We have evidence of these affordances in Lola and Mona's journal exchange. First, Lola's and Mona's journals are posted as pdf documents on Google Drive, preserving their use of color. The importance of color is mentioned in the comments shown in Figure 9. Lola commented that Mona's color-coding "is great," but suggested that a key be included as "a visual reference." Thus, Mona learned the importance of fine-grained attention to the needs of the learner struggling to understand mathematics. The use of color and its importance to the reader contributed to the work of the PSTs that would not have been possible in our original physical exchange of the black-and-white photocopied journals.

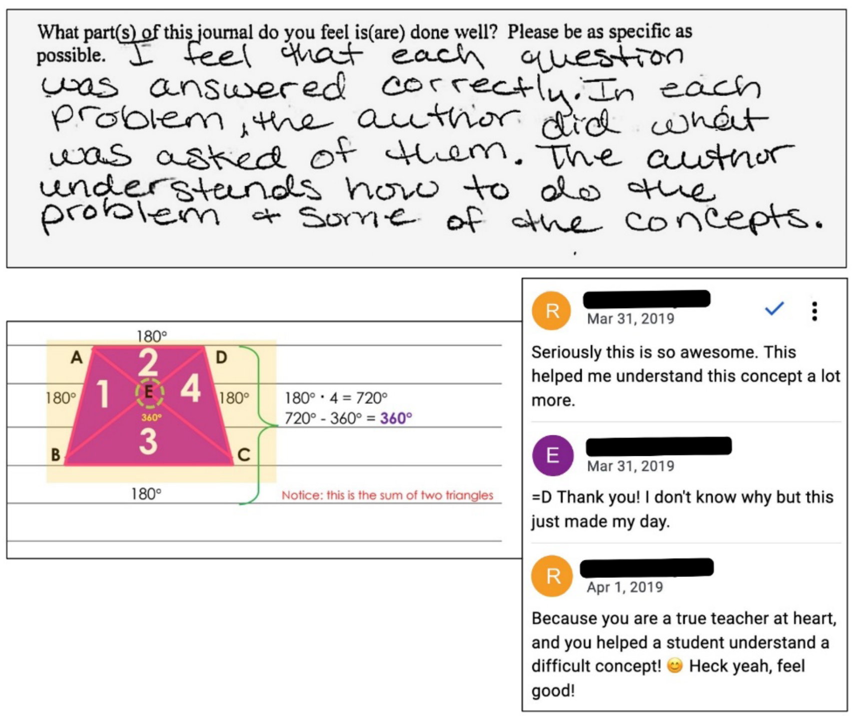
The second affordance of the digital platform is the opportunity to comment on specific parts of the journal by highlighting sections and commenting on those sections. For example, as seen in Figure 5, Mona addressed specific parts of Lola's work noting, "This is a really cool way to be able to reference the numbers in your table." She also prodded Lola to improve her work by commenting, "Maybe it would be beneficial to write out some of the equations that you used next to the table or talk a little bit about how you filled it in." In this comment Mona acknowledged Lola's work and encouraged her to develop it further by including more detail. The digital platform allowed Mona to clarify what "this" was by connecting the comment to a specific part of Lola's explanation, thus, making their interaction more effective.

The third affordance, seeing the mathematical work and comments in one document, was facilitated by the digital platform. When reading their partner's feedback, PSTs could immediately see their work and their partner's comments allowing for them to make connections from the comments back to their work in the moment. They did not have to move between different documents to find recommendations for improvement of their work.

The fourth affordance of the digital platform is the ability to comment briefly or extensively. PSTs could post brief comments of approval (e.g., "Nice work!"). They could also question the work, probe to learn more about the author's thinking, and engage in conversations with more extensive comments. Multiple back-and-forth dialog humanized and personalized the journal exchange process.

Figure 12 is taken from another pair of PSTs. It provides an example of how the digital exchange helped shift the process from an anonymous peer review of the work of "the author" to a more personal, collegial conversation. The author, Erica, expressed her gratitude for the reviewer's comment. The reviewer, Regina, said that Erica's explanation "…helped [her] understand the concept a lot more." Erica responded, "Thank you! I don't know why but this just made my day." Regina offered additional encouragement: "Because you are a true teacher at heart, and you helped a student understand a difficult concept! Heck yeah, feel good!"

Figure 12
Personalizing and Humanizing the Exchange: Physical Versus Digital Exchange



The fifth and sixth affordances capture the flexibility of the digital platform. Once posted, documents were always available in Google Docs. PSTs could access the documents when they were able to read and respond. Several times Mona and Lola commented and responded on the same day. However, as shown in Figure 11, there was a 2-day lag between Mona's comment and Lola's response. PSTs were able to comment and respond when they were ready, either quickly or after they had time to think through comments. Thus, the digital exchange provided the flexibility suggested in the literature (Singer & Zeni, 2004; Sutherland et al., 2010).

Implications for Teacher Education

The addition of CMC provided the opportunity for peers from different campuses to have conversations based on their mathematical writing. As one PST said,

> At first, I was hesitant with google docs, but I am much more comfortable with operating that now. Knowing how to respectfully make comments on the work and be face to face with the reviewee can be valuable. Technology can be super-efficient, time saving and neater as well.

The digital exchange activity produced valuable mathematical learning experiences that still retained a personal feel.

As our work progressed, we observed particularly rich interactions between journal partners and modified our practices with the goal of further developing their mathematical knowledge and their ability to communicate mathematics. The changes included refining the prompts to guide PSTs' reviews, maintaining review partners for consecutive semesters, introducing the journal exchange through exemplars, and building collaboration into the review phase. We consider each of these in the following sections.

Improving Prompts for Journal Reviews

Mathematics teacher educators should provide clear expectations for the journal reviews while creating a flexible environment to foster a collegial exchange. In our initial work, we instructed the PSTs to highlight and comment on the sections of their partner's journals that were well done as well as parts that could be improved. Many reviews focused on explanations, the use of representations, layout, and other formatting related issues (as can be seen in Table 2). We asked them to highlight specific mathematical ideas with the aim of guiding them to converse about the mathematics concepts as well (see initial prompts in previous sections).

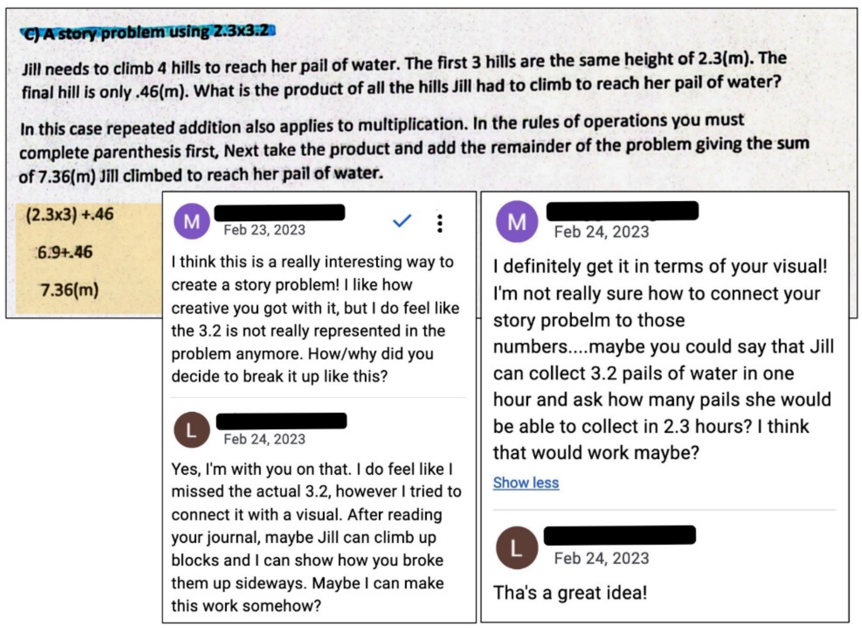
As previously mentioned, when students organically began to reply to their reviewers' comments, new opportunities to develop their mathematical knowledge became apparent. This result prompted us to revise our prompts to formalize this extra layer of conversation. When we prompted all authors to respond to their partner's comments and questions, we observed more in-depth mathematical dialogs with opportunities to rethink, revise, and clarify earlier explanations and representations.

Maintaining Review Partners for Consecutive Terms

When possible, we encouraged mathematics teacher educators to maintain review partners throughout multiple journal exchanges. When we began to use the digital platform, we randomly assigned partners for each exchange. We observed that some pairings fostered particularly rich interactions, but these relationships were often short-lived, as PSTs had different partners for subsequent journals. In later semesters, we maintained the same journal pairings and found these continued conversations were more meaningful.

Pairing Mona and Lola across two semesters demonstrated the value from such a sustained pairing. Figure 13 shows an exchange between Mona and Lola over representing decimal multiplication during the second semester of their work together. This interaction seemed to help Lola improve her word problem leading to the calculation of 2.3 x 3.2. Their ongoing partnership over two semesters appears to have fostered a respectful relationship that helped each partner develop their confidence and quality in their mathematical work.

Figure 13
Mona's Review of Lola's Work During Their Second Semester of Journal Exchanges

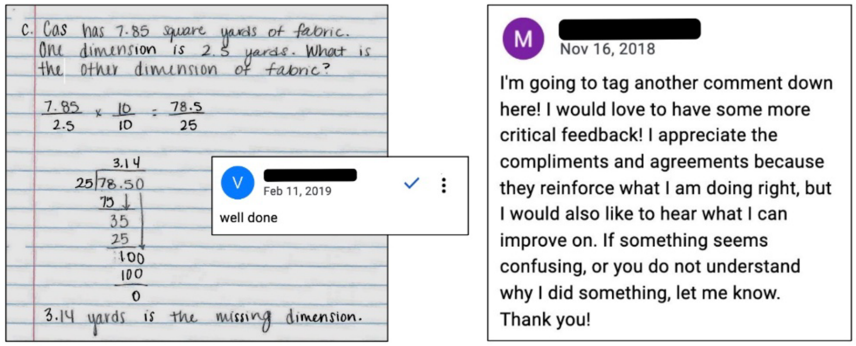


Introducing the Exchange with Exemplars

We recommend that teacher educators share examples of productive and nonproductive journal reviews with PSTs. The PSTs come to understand quickly that pleasantries, while nice to hear, might not be helpful as they seek to improve their teaching practice. It is more helpful when the reviewer asks targeted questions or provides specific information about the positive aspects of the mathematical explanation, rather than simply commenting, "Well done." PSTs realized this as well, and, in many cases, requested more critical feedback (see Figure 14).

In an early example of an electronic exchange, Melinda, after receiving many pleasantries, posted a comment at the end of her own journal. She wrote, "I appreciate the compliments and agreements because they reinforce what I am doing right, but I would also like to hear what I can improve on." The instructors of the course then began to urge reviewers to do this right from the start and offer specific strategies for raising the level of discourse in the reviews.

Figure 14
Pleasantries Versus Critical Feedback



Through reviewing and being reviewed, PSTs learn how to comment on their partner's work to create more meaningful and effective journal reviews. PSTs need encouragement to "find their voice" within the review process. Therefore, it is important to convey to the PSTs that they have flexibility and are encouraged to share their perspective on the work, bringing their personal touch to the review.

Collaborating on the Journal Review

There were semesters when the two institutions' class sizes were very different. Often, multiple university students had to be partnered with one college student or vice versa. Initially, the PSTs reviewed the single author's journal individually. An exchange between Uma and her partners, Melissa and Elise, exemplified how separate reviewers frequently chose to comment on the same features of the journal (see Figure 15). Both Melissa and Elise lauded Uma on her representation of place values for decimal multiplication.

A similar situation is evident in another exchange from that semester, where two reviewers commented separately (see Figure 16). Both Eunice and Llandra appreciated that the author, Sandra, used side notes to help clarify her explanation. Although this repetition might help to draw attention to a particular point done well or needing work, the instructor of the course decided to have the two reviewers conduct their work jointly, in class, for the second exchange of that semester.

Figure 17 shows a pair of comments from the second exchange of the semester, in which Eunice and Llandra reviewed Sandra's journal together. Notice that Eunice alerted Sandra to the collaborative commenting by writing both their names as the first line of their comment (here redacted). We found that collaborating in class led to more focused, less redundant, and deeper comments. Thus, when possible, we recommend having multiple reviewers collaborate on the review component. This adds another layer of collaboration that PSTs may value and carry with them into the field.

Figure 15
Melissa's and Elise's Comments on Uma's Explanation of Decimal Point Placement

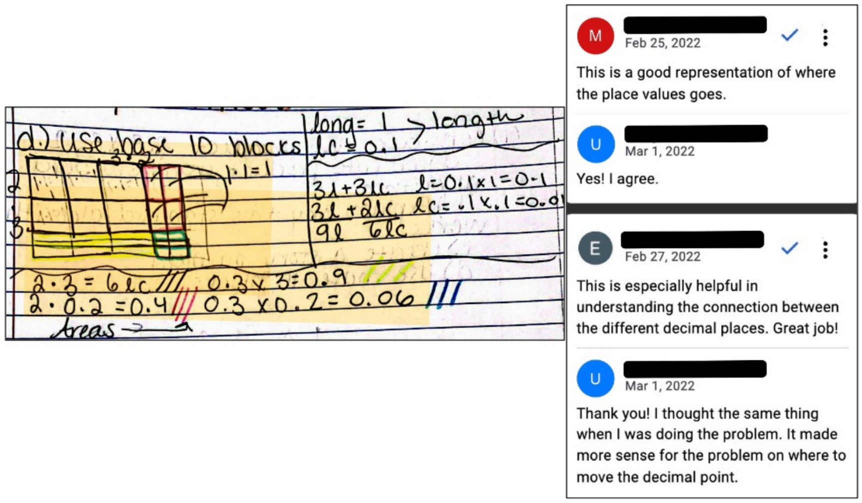
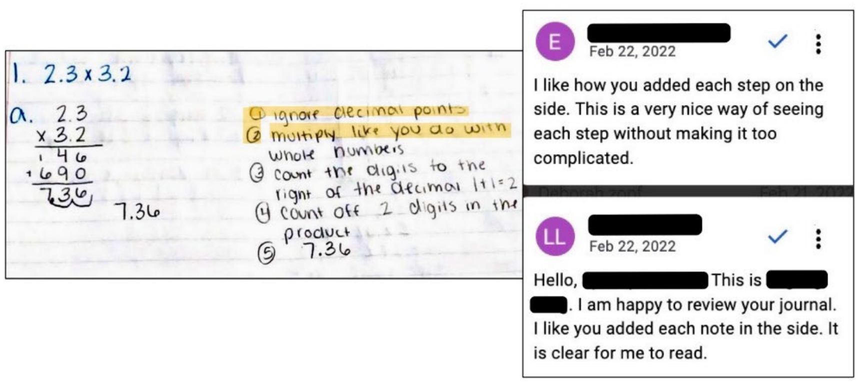
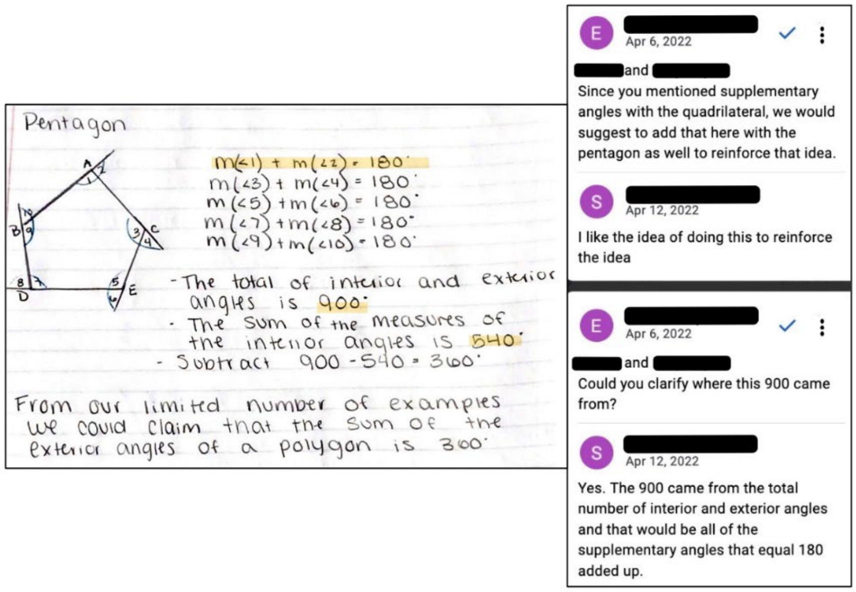


Figure 16
Eunice's and Llandra's Comments on Sandra's Decimal Multiplication Journal



Having discussed how the affordances of the Google Doc platform supported the interinstitutional conversations presented here, we note that other platforms may be better suited for other contexts. Shared-document platforms with commenting conventions similar to Google Docs, such as Perusall and Microsoft OneDrive, abound. In addition, mathematics teacher educators without contacts at other institutions can facilitate intramural exchanges between students from other sections of the class at their own institution. The OTLs within the digital Exchange Phase are robust enough to endure such context-specific modifications.

Figure 17
Eunice's and Llandra's Collaborative Comments on Sandra's Second Journal



Concluding Thoughts

The AMTE (2017) expected that "well-prepared beginners encourage students to communicate their reasoning, critique the reasoning of others, and develop arguments through discourse and mathematical writing" (p. 77). To support their future students to do this effectively, PSTs need to experience communicating their own mathematical thinking and reasoning via writing. Having a peer review their writing adds another layer to this process. With it, PSTs now write to make their mathematical thinking visible and comprehensible for someone else.

With the capabilities of a CMC environment, this review process can become a rich mathematical dialog. The challenges in implementing this work, however, must be acknowledged. Mathematics educators must be willing to devote considerable time and curricular space to the APEX Cycle. Each week, on average, 1 out of the 4 contact hours was devoted to the Analysis and Presentation phases of the cycle. Reading and grading the journal entries for classes with 20 PSTs, while often exciting and inspiring, requires more attention and time from the instructor than other assessment forms. Organizing and monitoring the Google Drive and nudging PSTs who have not uploaded their work or offered feedback to their partner are also time consuming.

Equally important, mathematics educators should value the collaboration and exploration inherent in the cycle and seek to foster an appreciation of this pedagogy in their PSTs. Even with these challenges of time and coordination, based on the evidence we have presented here, this effort is worthwhile. Many PSTs shared that it was time well spent. One PST commented in an unsolicited note,

> I'm messaging you to let you know I actually really enjoyed doing the ... exchange. I loved how we were able to send our sheets to others and reply to them, with them replying back in a matter of minutes/hours. It was a great experience. My *friend* [emphasis added] from [the other school] was very nice and her feedback was great.

Another, recognizing the growth in her mathematical knowledge, said,

> Having someone else evaluate my work and provide suggestions for improvement allows me to see someone else's way of thinking and may be a way to get new ideas I have never thought of before. In the future, going beyond comparing what I wrote to my peers, I should think deeper about the content and maybe look at important concepts that maybe even I missed in my own journal to help my peer and provide a perspective they might not have seen.

Other PSTs will be well-served to have similar opportunities to deepen their mathematical knowledge through the APEX Cycle, particularly through the interinstitutional conversations made possible by the computer-mediated exchange of their mathematical writing. Although the PSTs in this case were undergraduates in traditional teacher education programs, teacher education students in nontraditional programs may also benefit from this experience. Future studies implementing this for PSTs along different certification pathways could be particularly valuable to the field.

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Appendix
Journal Exchange and Review

You will again be reviewing a journal written by a fellow student who is studying mathematics with the goal of becoming a teacher. Most of the pairings are the same as they were during the first review. As before, please treat this work with respect, and comment thoughtfully. **Constructive comments that go deeper than complements or pleasantries can help the writers improve their work and become more knowledgeable about mathematics and ways to communicate it**.

Just like the first review, you will place comments at specific points in the write-up. **If you need a reminder of the procedures for leaving comment within the Google Drive, please see the last page.**

Please use the following questions to guide your work. You may, of course, go beyond these suggestions.

What part(s) of this explanation do you feel is(are) done well? (Please be as specific as possible).

What part(s) of this explanation do you feel could be improved? (Please offer specific suggestions for improvement).

What are the important mathematical ideas that you see used?

Are these ideas used correctly?

Are there ideas and/or representations that you feel deserve more explanation or are missing? If so, what would you suggest adding?

PLEASE FEEL FREE TO ASK QUESTIONS ABOUT THE WORK/EXPLANATION.

Adding Comments on Google Drive

Click on the link in the message that you received. This will take you to the Google Drive containing all of the WSU and HFC journals.

If you have a Google account, you can log in at this point. One benefit of logging in is that the comments that you leave will automatically include your account name (it's a bit more personal). If you do not have a Google account, please create one now.

Open the file that you wish to review **inside Google Drive (please do not open it with another application or download it to your computer)**, by double clicking the file name.

Review the document and post comments in the appropriate areas.

To post a comment, click the "Add a comment" icon at the top (which looks like a dialogue bubble with a + inside), and it will tell you to highlight an area to comment on.

Click and drag on that area (a particular word, sentence, figure, or blank space), and the comment box pops up.

When you finish your comment, click the blue "Comment" button.

When you are finished adding all of your comments, you can simply close the document by clicking on the white left arrow to the left of the file name. There is no need to save as Google Drive does this automatically.

If you experience any trouble, please feel free to contact your instruct