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Using Virtual Reality During Remote Learning to Change the Way Teachers Think About Geometry, Collaboration, and Technology

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Extended reality experiences for mathematics teachers can allow them to understand both pedagogy and the mathematics itself in new ways. In this study, the authors explored a virtual reality simulation for learning about geometric shapes, where teachers could engage in joint, shared manipulation of holograms in three dimensions. The authors examined what teachers saw as the affordances and limitations of this activity, as well as how the activity transformed their understanding of extended reality, through three cases of groups of teachers. Important themes emerged related to engagement, tangibility, collaboration, and dynamicity of the virtual reality environment, as well as serious concerns relating to space, cost, and physical issues. Implications are discussed for training teachers and teacher educators to implement embodied approaches to mathematics instruction.

Extended reality (XR) is an emerging avenue for teacher training and professional development. The term XR incorporates virtual reality (VR), augmented reality (AR), and mixed reality. In mathematics teacher preparation, XR technologies may be powerful for allowing teachers to interact with and understand math ideas in new ways and to consider new ways of teaching.

AR and VR platforms now support shared, immersive experiences that enable learners to engage directly, physically, and perceptually with mathematical objects. These new forms of AR/VR technology – which we call shared holographic AR/VR (shAR) – enable multiple learners to manipulate the same mathematical objects represented as holograms projected in a joint three-dimensional collaborative space in front of them, using intuitive hand gestures.

XR technologies are becoming more prevalent and affordable given their availability on smartphones and plummeting costs of headsets (Fransson et al., 2020). The price of a 6-degree VR head mounted display has decreased from \$599 (Oculus Rift in 2016) to \$299 (Oculus Quest 2 in 2021). The VR market in education is expected to grow at a rate of 42.5% between 2021 and 2026, with North America being the largest and fastest growing market (Mordor Intelligence, 2021).

Recent advances related to shAR are significant because shared, immersive manipulation of 3D objects has not previously been possible and can bring together the affordances of physical and virtual learning (Bujak et al., 2013). ShAR has the powerful dynamicity and immediate feedback of virtual manipulatives and simulations, combined with the gestural interface and 3D nature of physical objects, leveraging knowledge that is gestural, perceptual, and action-based in nature (McNeil, 2008). These platforms allow new possibilities for collaboration because learners can interact and embody concepts in a coordinated way (Lindgren & Johnson-Glenberg, 2013).

The full implications of shAR for math teacher professional development have yet to be explored. This article discusses a prototype shAR system for collaborative manipulation of holograms of geometric shapes. In the study reported here, in-service educators worked together using their hands to resize, move, and transform the geometric shapes (e.g., a cylinder or a triangle) in three dimensions, while exploring geometric conjectures and properties. The purpose of these activities was to change the way teachers thought about three-dimensional geometry, XR technologies, and collaboration and to help teachers imagine how such technologies might become relevant to their future classrooms.

Our long-term goal is to scale up the use of shAR for geometry learning into many classrooms, and to understand teacher thinking about these technologies. These outcomes will be key to designing activities and systems of training and infrastructure for shAR that are most useful to and relevant.

Theoretical Framework

Embodied Learning and Gesture

Theories of embodied learning highlight the ways all mathematical thinking is embodied – tied to perception, action, spatial systems, and physical motions like gestures. AR and VR technologies have incredible potential to leverage embodied approaches to learning and instruction. Johnson-Glenberg et al.'s (2014) taxonomy names three dimensions key

to the effectiveness of embodied learning interventions in educational settings: (a) motoric engagement, or the degree to which the body and the sensori-motor system are engaged in the learning process, (b) gestural congruency, or the degree to which the nature and types of gestures are relevant and map directly to the concepts to be learned, and (c) the degree to which the learner perceives being completely immersed in the XR environment.

ShAR technologies allow for these criteria to be met in ways not previously possible. Motoric engagement in shAR is high, as learners can gesture to modify objects and walk around and within objects. Gestural congruency in shAR is high, as advanced hand tracking allows for stretching gestures to enlarge shapes, rotation gestures to turn shapes, and so forth. Learners can become immersed using wireless headsets that layer math representations onto their environment (AR) or that create a new virtual environment (VR).

Embodied learning research in mathematics also highlights the importance of actions that are dynamic or transformational (Abrahamson & Sánchez-García, 2016; Fischer et al., 2015; Nathan et al., in press; Pier et al., 2019; Smith et al., 2014; Weitnauer et al., 2016) — when learners can change and manipulate mathematical objects through action and observe the effects.

Gestures — the spontaneous arm and hand movements that speakers produce when communicating — have been the subject of observational and intervention research because of their relationship with thinking, social cuing, and cognitive development (Goldin-Meadow, 2005). Gestures are a key way in which mathematics learning can become embodied. Alibali and Nathan (2012) found converging evidence that representational gestures, those depictive of objects and processes, exhibited students' mental simulations of actions, perception, and conceptual metaphors. During collaboration, gestures operate synchronously with speech, acting as a mechanism to create cohesion and bind conversational elements (Enyedy, 2005; Koschmann & LaBarron, 2002).

Walkington et al. (2019) documented how gestures can be used for collaboration in mathematics as students work in small groups — from students echoing or mirroring each other's gestures, to students building on each other's gestures with their own gestures, to multiple students jointly gesturing together to form a mathematical object. A key insight from this research was that students gesture in dramatically different ways when they are collaborating with others versus when they are working individually. In this way, cognition can become *distributed* across learners (Goodwin, 1995; Hutchins, 1995; Lave, 1988) in an embodied manner, where learners are using their actions, perceptions, and movements in concert with each other to advance their mathematical understanding and communication.

Our epistemological stance of embodied learning was the foundation of both the design of our VR environment and of the way in which data were collected and analyzed in the present study. In terms of design, our embodied cognition framework led to an environment where learners

could perform dynamic actions on mathematical objects using intuitive hand gestures and where they could work together in a collaborative virtual space and coordinate their actions in real time using speech, movement, and gesture.

We collected data in such a way that all these multimodal streams could be carefully analyzed and used to contextualize the statements teachers would later make when reflecting on their experiences. These ideas of embodiment, dynamicity, and collaboration also guided our development of coding categories for understanding how teachers conceptualized the environment, although we were open to including ideas outside of this framework from the teachers as well.

Embodied Learning and Gesture in Mathematics Teaching and Teacher Education

Studies suggest that teachers' gestures during instruction can enhance student learning and meaning making (Alibali et al., 2013; Singer & Goldin-Meadow, 2005; Valenzo et al., 2003). Teachers can use pointing gestures to direct learners' attention to particular objects or portions of a representation or display. They can use representational gestures directly to formulate mathematical objects or ideas with their hands (Alibali & Nathan, 2012). Teachers can use gesture to create cohesion across different representations, settings, and learning events, creating links over time, space, and materials (Nathan et al., 2017; Walkington et al., 2014).

Professional development (PD) focusing on gestures and embodied learning may be beneficial for enhancing teachers' ability to promote students' mathematical reasoning. When teachers engage in PD on new strategies, the PD must "incorporate hands-on, experiential learning opportunities, that are embedded in authentic contexts in which teachers can thoroughly connect with the new strategies" (Richards & Skolits, 2009, p. 42). PD for mathematics teachers that involves play or action in the physical or virtual world can be especially powerful for training teachers to implement embodied learning (Abrahamson et al., 2020).

Teachers implementing embodied technologies can learn strategies to support students' multimodality, including being aware of students' gestures and actions, summarizing and reflecting students' ideas using gesture and action, and coconstructing ideas with students using gesture and action (Flood et al., 2020). Abrahamson et al. (2020) described how training preservice teachers in using embodied approaches to mathematics instruction can allow them to learn about student-centered, experiential learning strategies. Additionally, training in-service teachers to gesture in ways that connect ideas in mathematics can improve student learning (Alibali et al., 2013).

Extended Reality in Preservice and In-Service Teacher Education

VR is a fully immersive 3D multimedia environment where users can interact with a computer generated world (Milgram & Kishino, 1994; Onyesolu & Eze, 2011). Immersion, flexible control, and representational

fidelity are the technological characteristics of VR environments (Makransky & Petersen, 2021). VR technology can be used to provide immersive, interactive science, technology, engineering, and mathematics (STEM) learning experiences (e.g., Bennie et al., 2019; Dinis et al., 2017; Huang et al., 2021; Makransky et al., 2019; Putman & Id-Deen, 2019; Shi et al., 2019; Simonetti et al., 2020; Trentsios et al., 2020), which in many cases could be relevant to teachers as they engage in professional learning to deepen their own STEM understanding.

Only a small number of studies, however, have explored XR to train teachers. A recent systematic review of the efficacy of VR in teacher education included only seven studies (Billingsley et al., 2019) and found positive outcomes of VR on teacher learning and attitudes, with only one included study focused on teacher content area learning. One element of VR in teacher education that needs more research is the role of social interaction (Araiza-Alba et al., 2020), particularly as teachers like to learn in collaboration with their peers (Ripka et al., 2020).

A weakness of using head-mounted displays, in general, is that teachers cannot always see what is happening in students' or collaborators' environments. More research is also needed on how VR can be pedagogically justified (i.e., used for purposes where VR is needed and not merely for play) and aligned to the curriculum (Fransson et al., 2020).

One of the most frequently mentioned contributions of XR in preservice and in-service teacher education is supporting teachers to develop their pedagogical and classroom management skills in simulated environments (Dalinger et al., 2020; Dawson & Lignugaris/Kraft, 2017; Stavroulia & Lanitis, 2017). These experiences help teachers identify areas in need of professional growth and enhance their confidence in teaching through repeated practice.

VR can also be used to enhance teachers' professional noticing. Ferdig and Kosko (2020) explored whether using 360-degree videos in case studies could enhance preservice teachers' attention in a classroom context. Participants were assigned to watch a video about students learning the Commutative Property of Multiplication. These researchers found that watching 360-degree videos with the VR headset trained teachers' perceptual capacity and noticing. In addition, in-service and preservice teachers can be trained to develop simple AR and VR applications to support their teaching (Sáez-López et al., 2020; Weissblueth & Nissim, 2018). This type of activity can enhance teachers' creativity, innovation, and self-efficacy and foster their social and emotional learning.

Other research has explored how teachers view AR/VR activities implemented in their classes (Dunleavy et al., 2009). The experience can be highly motivating and facilitative of collaboration and interdependence. Teachers also experienced hardware and software issues, high logistical support and management needs, and potential threats to physical safety. These findings align with other research in which teachers have recognized the economic and technological difficulty of using VR, the initial learning curve, and the importance of PD (Fransson et al., 2020). These same themes could be mirrored in more contemporary uses of XR technologies, such as the use of shAR technologies.

Schenck et al. (in press) explored teachers' embodied learning practices when playing a motion capture XR game for geometry. They found that teachers' use of gestures when solving game problems was associated with their beliefs about the usefulness of gestures for teaching and learning. Teachers' gestures, particularly their gestures that were collaborative, were associated with more successful problem-solving.

Walkington et al. (2019) also found that before the game experience, teachers conceptualized the body in mathematics learning to help students memorize mathematical facts (e.g., "Make a crocodile with your arms to eat the bigger number.") and as a physical/psychological tool (e.g., "Use your feet to measure," "Use your hands to handle manipulatives," or asserting that it's good simply to "get students' blood pumping").

After the experience, teachers were more likely to see the body as helping understand mathematical ideas (e.g., "Use your arms to show the angle in a triangle growing and observe what happens to the opposite side.") and as a conceptual tool to understand mathematical relationships (e.g., "Use your body as a manipulative to test geometric relationships."). While this study provided interesting implications for research on motion capture, it did not directly address AR and VR.

Research Questions

Prior research has explored the use of XR to allow teachers to gain professional skills in simulated classroom settings, including professional noticing (e.g., Ferdig & Kosko, 2020). Some prior work has also looked at teachers' reactions to XR innovations implemented with their students (e.g., Dunleavy et al., 2009). Little prior work, however, has investigated ways the novel affordances of XR technologies can change the ways teachers think about teaching the content itself (here, mathematics) and change their thinking about what it means to collaborate around this content using technology.

Further, as shAR is relatively new, the field does not yet have a detailed view of the mechanisms for getting teachers to implement these kinds of innovations at scale and their perceptions of key limitations and opportunities. We, thus, posed two research questions in the present study:

1. What do teachers see as the affordances and limitations of shAR technology for learning mathematics?
2. How do teachers' ideas about shAR for math learning change after an immersive shAR experience?

Method

Participants and Context

Nine female in-service teachers, who used shAR Geometry Simulation Environment (shAR GSE) in groups with an instructor present in their virtual room, participated in this study. The teachers were enrolled in a synchronous virtual education course focused on geometry for current

math teachers at a university in the United States; all teachers in the course consented to participate in the study. The course was taught virtually due to the COVID-19 pandemic. The teachers checked out Oculus Quest goggles to participate and joined from their individual homes, where they were instructed to have a clear area to play.

Teachers had an average of 3.9 years of teaching experience, with three teachers who taught Grades 5-6 mathematics, five teachers who taught Grades 7-10 mathematics, and one teacher who worked as a technology lead for Grades 4-6. Three teachers identified as Asian, six as White, and one as Hispanic (teachers could choose multiple categories). The teachers were a mix of experienced Montessori teachers who were part of a Master's program in Montessori instruction and Teach for America Corps teachers in their first 1-3 years in the classroom, teaching in urban schools where a high proportion of students were experiencing poverty and were English language learners. Montessori schools are common in the United States, and their instructional approach is based on the work of Dr. Maria Montessori, an Italian physician. The approach leverages multisensory learning, inquiry, and student-led self-paced instruction (American Montessori Society, 2021).

Environment: The shAR GSE

Our shAR GSE functions in both AR and VR. In the present study, we used Oculus Quest VR goggles. The simulation is accessed as an app installed on the goggles. Users enter their name and select from a list of classes to join (opened by instructors). Each classroom holds up to six people.

When students join a classroom, if they are joining in VR, they appear as a generic head and torso with their name hovering overhead; the head and torso moves around the environment as they move. When they move their hands within view of the goggles, their hands are tracked and displayed in real time to the other users in the simulation through hand-tracking. Oculus Quest users must designate a play space of 6.5ft by 6.5ft to ensure they can move around in the virtual environment. Users can also hear each other's voices through the goggles.

The design of our environment was tightly linked to our theoretical framework. We wanted teachers to be able to manipulate the objects using gesture and to see each other's manipulations and hand gestures in real time to promote embodied communication. We wanted gestures, actions on objects, and speech to operate synchronously for teachers to jointly embody geometric ideas.

Tasks Given to Teachers

When the Geometry Simulation in shAR GSE launches, users see three shapes – two triangles and a cylinder. Teachers can touch the shapes with their hands to select them, turning them green. Once shapes are selected, they can grasp them and move them around. They can also be rotated or turned in the same manner. The triangle's three vertices have small cubes attached, which users can select and drag with their hands to transform the triangle's side lengths and angles. The cylinder has similar

manipulation points which allow its height and radius to be modified. The triangle's angle, side, and area measurements and the cylinder's radius, height, volume, and surface area are all displayed and updated automatically. All users see and interact with the same holograms, with the holograms updating based on collaborators' actions in real time.

Teachers move to the second stage of the simulation by placing the shapes in puzzle outlines. To do this, they must resize them appropriately. In the second stage, teachers can collaboratively manipulate a cube, a square pyramid, a sphere, a hexagonal prism, and a torus. Although a wide variety of mathematical tasks could be facilitated in the environment, Table 1 shows geometry tasks that were given to teachers during our study. We also had teachers participate in a projectile simulation. When the Projectile Simulation launches, teachers are in a virtual outside space and can shoot a projectile at different angles and with different levels of initial force using their hands. Teachers can move to a virtual indoor space, where they use the shooting mechanics to get the ball into hoops. These simulations were intended to teach concepts related to parabolas and angles. Only two of the four groups did the Projectile Simulation.

Table 1
VR geometry Tasks Given to Teachers

| Simulation | Tasks |
|------------------|---|
| Geometry Stage 1 | <ol style="list-style-type: none"> 1. One of your students conjectures that the volume of a cylinder changes by the same amount whether you increase the radius by 1 cm or increase the height by 1 cm. Do you think this conjecture is true or false? Why? Try it out with the cylinder in front of you. 2. Can you make it so the cylinder looks like a circle from both your viewpoint and your partner(s) viewpoint(s), at the same time? 3. One of your students conjectures that for all triangles, the largest side is always opposite from the biggest angle. Do you think this conjecture is true or false? Why? 4. Can you make the two triangles into a square, with one person controlling each triangle? |
| Geometry Stage 2 | <ol style="list-style-type: none"> 5. Can you and your partner(s) place your hands on as many faces of the cube as possible? Point your fingers to as many vertices of the cube as possible? Use your index finger and thumbs to cover as many edges as possible? How many vertices, faces, and edges does a cube have? 6. Choose two of the solids. Size them such that you have created two solids that you believe would have the same volume. How do you know they have approximately the same volume? |

Data Collection

Qualtrics Presurvey

The teachers completed a presurvey via Qualtrics that asked for demographic information including age, gender, race/ethnicity, language, educational background, current teaching context, and teaching experience. The presurvey asked also about attitudes toward gesture using the Teacher Attitudes About Gesture for Learning and Instruction (TAGLI) survey (Nathan et al., 2019), as gesture is an important element of the shAR GSE. Finally, it assessed the teachers' technology background using the Technology Autobiography (i.e., adapted from Racial Autobiography prompt in Singleton & Linton, 2005), which contained the following questions:

- What did you learn about technology growing up? What's the first time you remember using a digital tool? At home? At school?
- Who were the instrumental people in your exposure to technology? What specific experiences do you remember from your childhood related to technology?
- What is your favorite parts about using digital tools?
- What is the most annoying or worst part about using digital tools?
- What do you think the purpose of technology in schools is? What should it be?
- How do your teaching practices reflect this purpose?

The presurvey also asked teachers the following open-ended questions: (a) What is virtual reality (VR), and how might it be useful for teaching math? (b) What kinds of mathematical concepts from your grade level might VR be most appropriate or useful? One teacher chose not to answer the open-response questions.

Qualtrics Postsurvey

The postsurvey asked teachers five open-ended items, two of which were identical to items on the presurvey:

- What is virtual reality (VR), and how might it be useful for teaching math?
- What kinds of mathematical concepts from your grade level might VR be most appropriate or useful?
- What did you think of the activities we did in the VR environment during class?
- What are some of the strengths of VR technology for learning?
- What are some of the limitations of VR technology for learning?

This survey was distributed as homework after the VR class meeting.

Video From VR Environment

Another source of data was video of the teachers completing the tasks from Table 1 in the virtual environment. Video was recorded using the VR goggles recording feature; this feature malfunctioned for the first two groups of teachers, and as a result, only one of the groups was recorded, and one teacher did not have audio. Video was transcribed and entered into the NVivo software.

Video From End-of-Class Discussion on Zoom

Video was captured from an end-of-class discussion on Zoom, where students were engaged in discussion after leaving the VR environment. It was led by the professor, and a general structure for the discussion was planned in advance.

Data Analysis

Data from teacher surveys were extracted, and data from teacher VR sessions and postdiscussions were transcribed (including descriptions of gestures and actions) and entered into the NVivo Qualitative Analysis software. Videos were initially transcribed by a research assistant and then were further edited and polished by the first and third authors on this paper.

We first used thematic analyses techniques (Braun & Clarke, 2006) to analyze larger grain-size themes as the teachers responded to the pre- and postsurvey questions, as well as themes from the final live class discussion. In a spreadsheet, columns were made for each question asked on the surveys and the class postdiscussion. Then, teachers' answers to each question were pasted into a row by teacher. For the live class discussion questions, the transcripts of the teachers' responses were used and were put into the context of surrounding discussion when appropriate. Not all teachers responded to all questions during the postdiscussion.

Themes for coding the spreadsheet data were determined based on constant comparisons (Glaser & Strauss, 1967), where codes were formed by grouping and comparing teachers' statements through multiple iterations. These themes were compiled, with individual teachers' statements considered in the context of their background (including their technology autobiography), which were also present in the teacher's row in the spreadsheet to contextualize teacher responses. The coding categories and their incidence in the different data sources are provided in Appendix A.

We next looked at the video from the teacher VR sessions in NVivo, focusing on exchanges where the teachers were engaging in discussions in which their thinking was made clear and was related to our themes. From these analyses, we selected three illustrative episodes from the footage of teachers' work in the VR environment to zoom in on, conducting multimodal analysis (McNeill, 1992), with a detailed analysis of gesture, body movements and position, speech, and actions on virtual objects. We

chose two of these analyses to condense iteratively and make more succinct to present in the final article, due to length considerations.

Our final step was to divide the teachers into three groups or “cases,” based on their backgrounds and highlighted themes within each case. Our choice to highlight certain themes under some cases and not others does not necessarily mean that the themes did not also appear in other cases. We strategically chose themes to highlight in each case to keep the article succinct.

Researcher Positionality

The first author was a White female associate professor with a background in graduate mathematics and K-12 mathematics education. She was the conceptualizer of the shAR GSE and the instructor of the virtual course, so she got to know participants within the larger context of teaching the course. Although she emphasized collaboration, problem-solving, and dynamic mathematical simulations in the larger context of the course, she placed limited emphasis prior to this activity on embodiment and gesture, as the course was virtual.

The second author was a White female postdoctoral scholar with a background in teacher education and equity and social issues. She brought an outsider perspective to analysis of data in this study because her teaching experience was outside of the realm of mathematics education. Her predisposition was to look for prolepsis in action, or the ways that personal history related to an experience intersects with how one participates in a technology-rich environment, whether in person or in virtual spaces.

The third author is an Asian male postdoctoral scholar. His positionality is based on his prior work in VR in STEM subjects, and he paid attention to circumstances when actions and gestures improved users’ learning efficiency and whether this effect was sustainable in VR environments. The second and third authors participated in the research during the data analysis stage and after.

Results

To communicate the results, the nine teachers were organized into three clusters based on their background and the kinds of responses they gave to the VR environment.

Case 1: Experienced Montessori Teachers

The first three teachers whose experiences we discuss were the oldest (Cathy and Olivia in their 50s, and Val in her 30s) and most experienced teachers in the sample. All three also had experience and training as Montessori teachers, with Olivia and Cathy currently working at Montessori schools. Particularly for Cathy and Olivia, their experience with technology growing up was considerably different than that in schools today: “I remembered playing ping pong on an Atari, but I only started using computers in college on a big mainframe computer” stated Cathy. “I

learned BASIC and PASCAL in high school. I only used a typewriter in college,” Olivia said. Val had somewhat more recent experience, but said, “I grew up thinking technology was for people with money and was a tool to use for work.”

Affordances and Limitations of shAR Technology

Montessori schools often accentuated the use of tangible, hands-on manipulatives to make many mathematical ideas accessible. They also, to some degree, had traditionally disallowed the use of technology in the elementary grades. The COVID-19 pandemic had created unique challenges for Montessori teachers, as they both had to reconsider their stance on the use of technology and reconsider their use of hands-on manipulatives if they had students joining virtually.

I have half of my classes virtually and half of it is in person, and I feel like even at home ... they probably have more availability to actually physically hold things. You know it could just be find a cylinder in your house, and they go and grab a water bottle or find, you know, find a rectangular prism, and they go bring a book. It could be easier, but at school, because we're not supposed to be touching a whole bunch of stuff, I can show them stuff, but it's not the same. So I feel like with that [the simulation] to look at the volume, it was really cool, 'cause everybody, whether you were at home or at school, you could all still do that group work, right? They can all work together to try to figure it out. (Val)

These teachers seemed to find the use of VR in this manner to be consistent with the Montessori philosophy, which accentuates visual and hands-on mathematics. Cathy said, “It has the possibility of making lots of abstract ideas more concrete in a visual way for students.” Olivia was most positive about the experience, stating, “It is the best technology I have seen for learning. Changing the dimensions of shapes is even better than manipulatives.” Val said, “Being able to apply your imagination and create a visual of what is taking place when you are finding the volume or angles are an essential skill in really understanding geometry.” Olivia also made an observation about how transformative the encounter with VR had been for her:

It really felt like the like the younger kids must feel when they're exploring, because I didn't have access to my normal thing, which was writing down things, so it was like a different part of my brain almost. I think it probably felt like what the kids feel before they really understand the math.

For this group of teachers more than any of the others, the experience seemed to change how they saw technology, manipulatives, and math learning itself.

The teachers also acknowledged some important limitations of VR technology. Val said, “I'm glad we took a break, 'cause it was really starting to burn my eyes.” Cathy said, “Currently, there is still a lot of lag time; therefore, with children who are used to gaming with fast computers, they

might be bored quickly.” Olivia also made the sweeping comment, “I think many things have to work in harmony. The technology and the expertise must be so expensive. I can’t see this being accessible to many people.” Thus, these teachers recognized both the technical and logistical limitations of VR for learning.

Overall, our thematic analysis revealed that key affordances of VR technology for these Montessori-trained teachers included it being visual and hands-on. Some key limitations included VR being slow, technically difficult to use, physically difficult to use, and expensive.

How Did Teachers’ Ideas About shAR for Math Learning Change?

These three teachers expressed some doubts about the benefits of technology on the presurvey. Val wrote, “People are so attached to them [technology] and rely on them so much now. I feel like technology has changed the way people relate to each other and the world in a drastic way since I was a child.” Olivia stated, “I wonder if there are unintended consequences for health and development. I think it should be used for really unique purposes, like visualizing things outside children’s experience like atoms, cells, Ancient Rome, etc.” Cathy wrote, “It’s [technology is] good for graphic design and programming, but I think math should be done with pencils and paper.”

Prior to the VR experience, these teachers had a limited understanding of the use of VR for teaching and learning. When asked, “What is virtual reality, and how might it be useful for teaching math?” Cathy responded, “I don’t know; it’ll be interesting.” Val responded, “Perhaps make large and obscure objects seem more realistic. Looking at bridges and skyscrapers close up in a geometric way.” After the activities, Cathy said, “VR is a CGI used to simulation of ‘reality.’ Sometimes, textbook word problems are just not relatable to students, but using VR students visualize abstract concepts and help them better to understand the problems.” Val responded,

You are able to work with others and manipulate 3 dimensional shapes that are very hard to truly experience in a 2-dimensional world like a textbook or a computer screen. I think it could be really useful in helping create solid concrete ideas of what figures are, being able to truly immerse yourself in the math.

Taking these responses as well as the responses in the previous section together, our thematic analysis revealed that these Montessori-trained teachers seemed to gain an understanding of how VR could make mathematics more concrete, more visual, and more immersive through 3D representations.

Case 2: Novice High School Math Teachers

The next group of teachers were younger, with two in their 20s and one who was 30, all with 3 or fewer years of teaching experience. They all taught high school grade levels, with one also teaching 8th grade. They had more experience with technology. Vickey said, “We had a computer room

in our house where my sister and I would spend hours on, and we also started playing with gaming consoles early on. In school, I remember using the computer lab in Kindergarten...”

They acknowledged limitations of technology, like glitches, slowdowns, and technology negatively impacting people’s perceptions of themselves and the world. In addition, they seemed to understand the important role technology could play in schooling. Riley said, “The purpose of technology in schools is to utilize another way of teaching and give students more aspects and ways to see what they are learning and to be more hands on. I think it is on its way to enhancing learning, which is what it should be used for.”

Affordances and Limitations of shAR Technology

When describing affordances of VR, the high school teachers strongly accentuated affordances for collaboration between learners, particularly when compared to other virtual learning options. Vickey said,

It did simulate more of like being in person, just because I feel like it was a lot easier to collaborate because we saw exactly what everyone was doing... like sometimes on Zoom calls you're in a google doc, it's kind of hard to know what the other person is thinking.

Kelly described how

it felt more to the real thing, I guess, 'cause even though you weren't next to each other, it felt like you were next to someone. At some point a little too close, but it was cool to feel like it was like someone was there. And you were doing like an experiment together.

Riley agreed, saying,

Just the ability to like see their hands and, like, I don't know, see what they're doing with the shapes. We were just on Zoom. We can't do that. Just made it a lot easier to collaborate and understand what they were thinking.

Kelly also called attention to the power of VR tasks that require collaboration in an *explicit* way (e.g., Task 4 in Table 1) explaining,

When we were given the task like the triangle one, where we both each had our own shape to manipulate but then the second question had us collaborate to put the shapes together to create something new. I think that was, I enjoyed that better than the change the dimension of the cylinder, 'cause we were both trying to change it at the same time. So yeah, I think when we had our own manipulative and had an extension of like collaborate together and each of you make something together.

A transcript of Kelly and Riley doing this task is shown in Figure 1. This transcript shows the coordinated, embodied way in which this team interacted with the VR objects and the way they communicated using speech and actions with their virtual hands on virtual objects. These teachers also noted some limitations of VR technology, with Vickey citing issues related to behavior management and content coverage: “It is obviously not applicable to everything we learn, and there is always the possibility that students will not take the task seriously.” Kelly also brought up technical issues, “Some limitations include setting up the VR environment and overcoming technical issues. There would have to be a tech person in each class to trouble shoot issues such as the ones we encountered in the session.”

Figure 1
Riley and Kelly Exploring Task 4 in Table 1

1. Kelly: Yours is twenty-four. Let me change mine to twenty-four.
(Kelly moves corner of her triangle)



2. Riley: Should we both change it to a forty-five, forty-five, ninety?
3. Kelly: Oh yeah that's good.
(Each changing their triangle)



4. Riley: Forty-five oh.
5. Kelly: We want them all to be twenty-four, right? Cause your sides are twenty-four? What's twenty-four times twenty-four? So, it's twenty-four, twenty-four, and also the degree has to be like forty-five?
6. Kelly: Okay I'm going to flip mine so it's next to yours. How do I? Wait.



- (Riley flips her triangle)*
7. Riley: There you go.
(Kelly moves her triangle closer to Riley's triangle)



Overall, our thematic analysis revealed key affordances for these novice high school teachers included it being collaborative, with emphasis on tasks that require collaboration and its affordances for collaboration during virtual learning. Some key limitations included VR being technically difficult to use, not applicable to all areas, and causing classroom management issues.

How Did Teachers' Ideas About shAR for Math Learning Change?

The use of VR for collaboration also came out when considering how teachers' ideas about VR for learning shifted. Before engaging with the VR activities, Vickey described how "Virtual reality is a computer-generated simulation that allows the user to interact with an artificial environment. VR can be used to teach 3D math, and even to explain concepts on the x-y-z coordinate plane." After the activities, she added "It can be useful for teaching math when doing exploration activities and for groupwork." Similarly, before engaging with the VR experience, Riley described how "Virtual reality is a simulation experience. It could be useful to give real life experiences." Afterwards, she added "You are able to complete activities and collaborate with others in real time and what feels like with them... It is also a great way for students to work together." Overall, our thematic analysis revealed that these novice high school teachers gained knowledge about the power of *collaboration* in VR environments.

Case 3: Novice Middle School Math Teachers

The final group of three middle school teachers were in their 20s and had 2 or fewer years of teaching experience. These teachers had early experiences with technology. Jill said, "My teachers taught me how to use computers, how to type, how to search for things, while in third through fifth grade. I didn't start using technology for math until middle school." Nancy said, "I also had an iFly, which was a technological pen that had an educational book that talked to each other." These teachers identified glitches and the learning curve to use technology as barriers. Jill said, "Right now, technology is everything in schools. Teaching students how to be digital citizens is important for preparing them for the workforce."

Affordances and Limitations of shAR Technology

These novice middle school teachers accentuated the interactivity and dynamicity of the virtual objects. Jill said, "It was awesome to interact with many different 3D shapes. I liked being able to 'touch' and manipulate them. The exposure to this type of advanced technology was also fascinating to me." Nancy added, "They are real time manipulatives and are instant feedback and results on what would happen if you were to change things like the radius or side lengths." Melinda said, "I enjoyed the activities we did, especially the fact that we were able to manipulate the shapes. It reminded me of Desmos, but it was more intriguing in the VR setting."

The transcript in Figure 2 illustrates how VR objects are dynamic and interactive. It shows Jill and Melinda confronting the Geometry Stage 2-6 task in Table 1. They interacted with the objects by resizing them and discussing them using speech and gestures. The nature of the task capitalized on the highly dynamic and interactive properties of the virtual objects and how these properties facilitated experimentation.

These teachers also talked about student engagement in mathematics, which can be a particular focus for teachers at the middle school level.

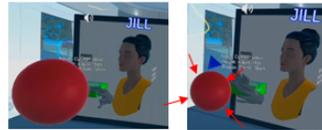
Nancy stated, “I think my students would really like it, 'cause I teach sixth graders, so like, the interactive part, I think they would really enjoy it.” Similarly, Melinda said, “Perhaps the biggest strength would be the experience because it increases engagement since it does not seem like work. VR almost makes learning feel as if it is a game for students.”

These teachers also talked about some of the limitations of VR technology. Melinda said, “As with anything new, with technology you should always expect some technological difficulties. I had some issues with finding a space that was big enough. Also a major limitation would be the price of the goggles.” Jill said, “One limitation for me was that the goggles required Wi-Fi and a decently large space. If a student wasn't able to access Wi-Fi or didn't have much open space at home, they may not be able to fully experience VR activities.” Nancy also mentioned that “they are expensive, and my campus would not be able to afford the goggles.”

Thematic analysis revealed key affordances of VR technology for these novice middle school teachers included it being engaging, interactive, and dynamic. Some key limitations included VR being technically difficult to use, requiring too much space, and too expensive.

Figure 2
Melinda and Jill Exploring Task 6 in Table 1

1. Jill: We can make the sphere smaller.
2. Melinda: Okay.
(Jill makes sphere smaller)



- (Instructor moves the donut into the joint space)*
3. Melinda: There's the donut.



- (Jill grabs the donut and makes it bigger)*
4. Jill: Okay how big?



5. Melinda: Bigger probably?
6. Jill: Yeah, right there.
(Jill makes the donut larger then releases her hands on the donut)



7. Instructor: Okay if you're going to pick one that you think might be a little bit bigger which one would it be?
8. Jill: The donut. The diameter is bigger.
9. Melinda: I don't know. Maybe the sphere.



10. Jill: I mean the donut, not only are we missing like the space in the middle, but it's also flatter.
(Jill moving hand and makes an open flat hand gesture roughly perpendicular to the torus, moving away and closer to show changes in front of the donut and the sphere to support the explanation the assertion that sphere was larger than the donut)

11. Melinda: Well kind of the same reason you're obviously missing a whole space, like in the middle.
(Melinda indicating the missing part of the donut with index finger)



How Did Teachers' Ideas About shAR for Math Learning Change?

Jill initially said VR would be useful for “Using manipulatives... discovery-based projects, seeing 3D objects” and it could be used in geometry for “understanding area, surface, area, and 3D shapes” and in algebra for “virtual manipulatives to solve equations.” Afterwards, she gave a slightly more expansive definition with some specific ideas for lessons:

It can be useful for teaching math in many ways. What seems most obvious to me is using it to observe and manipulate 3D figures or algebra tiles (essentially, just playing with tangible objects). But I think it could also be used for real-world simulations, such as a student going to a store and needing to calculate a discount on an item. I could see VR being useful for helping word problems come to life.

She also said,

For my seventh graders, a big topic is understanding circles and the relationship between radius, diameter, area, and circumference. When we were doing the cylinder conjecture together, it made me think of how a circle conjecture simulation (i.e., "A student argues that area and circumference will change by the same amount that the diameter changes") would work well.”

After the experience Jill had specific ideas about how VR could tie meaningfully to and lend itself to topics that she teaches.

Nancy also had refined her ideas about how specifically VR could be used. She initially defined VR as simply “simulated experiences” and said that it could be used for “Learning about area and volume, measuring and converting.” After the activities, she added that “it would be really helpful in dealing with manipulatives, like moving the angles and side lengths of geometric figures.”

Melinda was also able to cite some specific ideas for classroom experience after the activities:

The activities we actually did in the simulation would be a great example of what to use in my class. Later in the year we work on area and volume, and I think giving students the opportunity to play with the shapes and accomplish certain tasks with them would be beneficial. This would also be a great thing to use for the angles unit I touch on. Quite honestly, I feel as if there are endless options to use a VR experience.

Overall, our thematic analysis revealed that these novice middle school teachers gained knowledge about possibilities for teaching specific math concepts in VR environments.

Teaching Math Concepts With VR

We conducted one final supplementary analysis to answer the second research question, in which we examined changes in the way teachers answered the question, “What kinds of mathematical concepts from your grade level might VR be most appropriate or useful?” Responses were coded according to whether the application of VR to mathematics the teacher gave was static or dynamic.

Static applications involved using VR to observe specific math concepts, particularly ones that were visual in nature (e.g., shapes, graphs), without mention of students manipulating or changing those representations. An example was, “When analyzing geometric shapes, students could take a tour through a museum or building in order to better analyze and engage with such buildings.”

Dynamic applications involved using VR to enact transformation and change on mathematical representations. An example was as follows:

For my seventh graders, a big topic is understanding circles and the relationship between radius, diameter, area, and circumference. When we were doing the cylinder conjecture together, it made me think of how a circle conjecture simulation (i.e., “A student argues that area and circumference will change by the same amount that the diameter changes”) would work well.

Another teacher gave a dynamic response stating,

Geometry properties such as angles might be most useful since students could make conjectures about triangles, such as the congruency and similar triangle theorems. Also, this could be useful in dealing with geometric formulas. Seeing how changing one quantity affects the other could help students see how formulas intuitively make sense.

Overall, we found that while only two of the nine teachers gave dynamic applications prior to the VR experience, all nine teachers gave dynamic applications after the VR experience (see Table A3 in the Appendix for coding results).

Discussion

XR technologies have huge potential in mathematics teacher education, both to transform teachers’ pedagogy and to transform the way they think about mathematics itself. In the study described here, we investigated how immersive, collaborative geometry simulations where teachers could dynamically manipulate shapes and objects changed teachers’ views of VR for math education. We also explored what teachers perceived as both affordances and constraints of these novel technologies. Our investigation was framed using embodied learning (Wilson, 2002), and highlighting the importance of physical movements and gestures as a powerful element of AR/VR for math education (Johnson-Glenberg et al., 2014).

We centered the collaborative nature of the VR environment and the novel affordances it brings for student interaction, including collaborative forms of action and gesture (Walkington et al., 2019). Ideas of embodiment, gesture, and collaborative embodiment/gesture are often not highlighted in teacher education. Teacher education programs that expose teachers to shAR as a tool to teach students mathematics concepts are also rare. We thus expected that teachers' reactions would shed light on both the possibilities and limitations of shAR technology.

Affordances and Limitations of shAR Technology

Our analysis revealed themes for what teachers saw as the key affordances of shAR. The teachers identified the shAR system as being engaging. ShAR may be more motivating than manipulatives for older students, who may see traditional manipulatives as a “toy” (Bujak et al., 2013). The agency that comes from being able to act on the world in three dimensions is also considered a profound affordance of AR/VR (Johnson-Glenburg, 2018), and AR/VR experiences can, as a result, elicit surprise, curiosity (Bujak et al., 2013) and immersion (Johnson-Glenberg, 2018; Wu et al., 2013).

Teachers also identified the shAR system as being interactive and dynamic. Like a more traditional dynamic geometry system, the shAR allowed students to dynamically interact with objects and their properties to enact geometric relations and transformations. Like dynamic geometry systems, shAR can take precise measurements of objects which instantly update as the object is changed (Hollebrands, 2007). These affordances can allow a transition from reasoning about individual objects to reasoning about classes of objects.

The teachers further identified the shAR system as being visual and hands-on. Gestural congruency is a key affordance of shAR (Lindgren & Johnson-Glenberg, 2013), which was recognized by the teachers. Research suggests that learners may prefer interfaces where objects are manipulated via gestures, rather than buttons and menus (Zuckerman & Gal-Oz, 2013). However, more transparent interfaces can keep students from consciously recognizing they are engaging in mathematical operations like rotation (Sarama & Clements, 2009). The concrete nature of shAR objects may allow learners to better leverage prior knowledge and use new strategies (e.g., Goldstone & Son 2005). Students can also change their perspective by moving their body and are able to interact directly with 3D figures instead of 2D projections of 3D figures (Dimmel & Bock, 2019; Johnson-Glenburg, 2018).

Finally, teachers said that shAR had important affordances for collaboration. Unlike learning with screens, students do not have to transition between looking at a screen versus looking at their collaborators – all exist in the same space (Shelton & Hedley, 2004). Each collaborator in the environment has their own unique perspective and can control the simulation (Bujak et al., 2013; Johnson-Glenberg, 2018). Students can use gesture, speech, and actions on objects to coordinate their ideas and create mathematical meaning together (Walkington et al., 2019). Overall, the affordances teachers saw mapped relatively well to the key affordances of shAR outlined in the literature, and teachers saw the incredible potential of this technology.

Our analysis also revealed several themes in what teachers perceived as the limitations of VR. Some key limitations included VR being slow, technically difficult to use, physically difficult to use, requiring too much space, not applicable to all areas, classroom management issues, and too expensive. Many of these issues – such as cost, technical difficulty, classroom management, and speed – would apply to most new technologies being introduced into the classroom, including computers (Crossley & McNamara, 2016; Delgado et al., 2015; Islam et al., 2015). Some issues are unique to shAR, however. Specifically, the amount of space needed is a major issue that we did not foresee, particularly if VR is used rather than AR.

Simply using AR in classrooms might seem straightforward then, but AR goggles are far more expensive. ShAR GSE runs on the VR goggles Oculus Quest 2, which cost \$300 each, and the AR goggles Microsoft HoloLens 2, which cost \$3500 each. Physical difficulty with goggles may be a factor, such as strain on the head/neck, goggle adjustment issues, and motion sickness still being a problem for some (Jerald, 2015). Overall, the teachers identified some serious limitations with use of XR technologies at scale.

How Did Teachers' Ideas About shAR for Math Learning Change?

Our analyses suggest that teachers learned new ideas about ways technology could be integrated to make mathematics more hands-on and immersive, new ideas about how shAR collaboration could change students' interactions in the mathematics classroom, and new possibilities for teaching specific concepts through shAR. The case of the three Montessori teachers was particularly powerful for showing how shAR can change the way teachers think about technology for learning – from an avoidance of screens, to an understanding of using technology as an immersive 3D system to model the world dynamically.

The possibilities for collaboration in shAR are equally compelling, particularly given research on the importance of and power of collaborative forms of embodied learning (Walkington et al., 2019, 2021). Further, teachers' increased understanding of how math concepts could be modelled dynamically in VR from the beginning to the end of the experience was striking.

Finally, understanding which concepts in each curriculum are the “high leverage” math concepts that shAR would be most appropriate and powerful for is an important advance for teachers to make and for researchers to partner with teachers to understand. We do not yet see a world where every math concept is taught through shAR, but shAR could transform the way students think about some math concepts and better see mathematical ideas.

Implications

A major ongoing critique of teacher training in the U.S. is the separation of theory from classroom practice, as evidenced by the growing number of alternative and residency-based teacher education programs (Guha et al.,

2017). One way to encourage the bridging of this divide is to engage teachers at multiple stages of their careers in powerful XR experiences that provide opportunities for trying out research-based strategies that may otherwise require too much of an opportunity cost in their live classroom (e.g., practicing leading controversial issues or, in our case, learning about shAR to later support student inquiry in to similar XR contexts).

XR experiences can also allow learning to become embodied, distributed, and dynamic in new and powerful ways. This feature is especially important as XR technologies become increasingly accessible – for example, GeoGebra 3D is an AR technology that is free and works from any device with a camera that can run the app. It has powerful capabilities for engagement, dynamicity, and understanding and modelling of 3D objects, but is more limited in its affordances for collaboration and gesture. The strengths and weaknesses of XR technologies identified here, as conceptualized and taken up by teachers, can inform the development of the next generation of technologies that are designed to be readily adopted in and integrated into classrooms.

A growing community of teacher educators and teachers focuses on embodied instructional design (Nathan et al., 2019). XR can allow teachers to directly experience the power of embodiment and gesture for learning in ways not previously possible. Building on the work of embodiment in children's learning and ongoing studies, educators can begin to see the opportunities for increasing pre- and in-service teacher experience with the use of digital technologies for developing knowledge rather than being receptacles of content knowledge to funnel into students.

When teachers experience high-quality pedagogy made possible by XR environments in their teacher education programs and professional development, they are more able to identify the opportunities and challenges of active technology use over passive content consumption to continue to improve their own use of technology in their classrooms. In this way, our analyses suggests that XR and other technology experiences might be particularly important for disrupting and changing math teacher beliefs and practices. Given that mathematics and mathematics education are often viewed as subject areas heavily based on tradition that do not substantially grow and change, such technologies may have a unique place in math teacher education.

Future studies in this area could focus on figuring out which concepts across different disciplines benefit most from being taught through XR (e.g., geometric transformations and geometric nets), to determine where to invest efficiently the considerable amount of effort required to use these approaches. They could also focus on the training and institutional systems needed for teachers to feel comfortable trying out XR experiences in their classrooms. They could start with small experiences during teacher training, like the one presented here, and then give teachers increasingly more freedom and responsibility for implementing XR in their contexts. As teachers using XR in K-12 classrooms is in the early stages of its existence, qualitative research that examines rich patterns of interactions and teachers' reflections would be particularly important data sources.

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Appendix A Coding of Responses

Incidence of coding categories in different coded survey items and discussion transcripts, given as the number of teachers out of the sample of 9 teachers who mentioned this theme.

| Coding Category | Presurvey: What is virtual reality (VR), and how might it be useful for teaching math? | Postsurvey: What is virtual reality (VR), and how might it be useful for teaching math? | Postsurvey: What are some of the strengths of VR technology for learning? | Live Classroom Discussion Transcripts |
|------------------------|---|--|--|---------------------------------------|
| Dynamic | 1 | 5 | 6 | 7 |
| Collaborative | 0 | 4 | 5 | 5 |
| Immersive | 3 | 3 | 1 | 3 |
| Engaging | 0 | 0 | 2 | 3 |
| Visual | 1 | 1 | 4 | 2 |
| Hands-On | 3 | 5 | 6 | 3 |
| Concrete | 0 | 2 | 1 | 1 |
| 3D | 2 | 5 | 1 | 0 |
| Specific Math Concepts | 0 | 3 | 0 | 0 |

| Coding Category | Postsurvey: What are some of the limitations of VR technology for learning? | Live Classroom Discussion Transcripts |
|--------------------------------------|---|---------------------------------------|
| Expensive | 5 | 0 |
| Technically difficult to use | 4 | 3 |
| Physically difficult to use | 1 | 3 |
| Issues with amount of space required | 3 | 0 |

| Coding Category | Postsurvey: What are some of the limitations of VR technology for learning? | Live Classroom Discussion Transcripts |
|--|---|---------------------------------------|
| Slow | 1 | 0 |
| Not applicable to all areas/situations | 2 | 0 |
| Classroom management | 1 | 0 |
| Wi-Fi access | 1 | 0 |

| Coding Category | Presurvey: What kinds of mathematical concepts from your grade level might VR be most appropriate or useful for? | Postsurvey: What kinds of mathematical concepts from your grade level might VR be most appropriate or useful for? |
|---|--|---|
| Static use of math technology to teach concepts (e.g., observing, illustrating, describing) | 6 | 0 |
| Dynamic use of math technology to teach concepts (e.g., manipulating, changing, transforming) | 0 | 6 |
| Combination of static and dynamic applications given | 2 | 3 |