

Professional Development Supporting Teachers' Implementation of Virtual Manipulatives

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Supporting teachers' implementations of technology in the classroom is a critical and longstanding issue in mathematics education. As access to various technology resources grows, a need exists for professional development opportunities that prepare teachers to integrate technology effectively to support students' mathematical learning opportunities. Virtual manipulatives (VMs) are one technology tool receiving increased attention. Despite the benefits to student learning, secondary mathematics teachers use VMs less frequently than elementary teachers. Therefore, this study investigated a professional development opportunity aimed at supporting middle and high school mathematics teachers' implementation of VMs. Findings indicate two tools (a repository of resources and a task analysis framework) supported teachers as they prepared to implement VMs and tasks. Additionally, teachers were further supported via time for active learning (teachers interacting with VMs related to their upcoming instructional units) and collaborative planning.

A critical and longstanding issue in mathematics education relates to supporting teachers' implementations of technology in the classroom. Professional organizations (e.g., National Council of Teachers of Mathematics [NCTM], the International Society for Technology and Education [ISTE], and Association of Mathematics Teacher Educators [AMTE]) have advocated for supporting teachers to use technology effectively. However, teachers often claim that they are not prepared to use technology effectively in their instruction (Albion et al., 2015). As access to various technology resources grows, a need exists for professional development (PD) opportunities that prepare teachers to integrate technology effectively in ways that support students' mathematical learning opportunities (Driskell et al., 2016).

Effectively using technology means teaching *with* not *near* technology. In this study, teaching *with* technology entailed teachers using technology to promote opportunities for students to develop conceptual understanding through reflection and communication (Hiebert et al., 1997). Additionally, it meant supporting students in connecting multiple representations (NCTM, 2014). On the other hand, teaching *near* technology meant using technology in a manner that did not promote opportunities for students to communicate, reflect, and connect mathematical representations. Teaching *near* technology, or using technology merely as an attention grabber, is a misuse of technology (Suh, 2016).

Teaching mathematics *with* technology, requires a deeper knowledge of mathematics and technology, as well as of the ways in which teaching with technology transforms mathematics instruction (Wilson, 2008). Furthermore, Wilson suggested that teachers need to learn how to distinguish between mundane uses of technology (teaching *near* technology) and powerful instructional uses of technology.

Often, PD is considered an integral component to teacher learning when changes to instructional practices and knowledge are sought (Driskell et al., 2016). Yet, minimal studies exist investigating PD opportunities aimed at supporting teachers' transition toward teaching *with* technology.

Therefore, this study investigated a PD opportunity aimed at supporting middle and high school mathematics teachers' implementation of virtual manipulatives (VMs). Although mathematics teachers have been implementing VMs more frequently (Moyer-Packenham et al., 2008), this frequency is known to decrease from kindergarten through eighth grade (Moyer-Packenham et al., 2013). The following are examples of VMs used by teachers during the PD:

Addition of Polynomials from ExploreLearning

<https://drive.google.com/file/d/1Zg2SAICVvo9zV15S4mwOE5AVmh9x3mPs/view?usp=sharing>

Cubes from Illuminations

<https://drive.google.com/file/d/1shYLesUlgTOlh7hewH7xQrhrs5j2R9He/view?usp=sharing>;

Derivative Sketch from Flash & Math

https://drive.google.com/file/d/1Nex_3vYHQBMxWqiItxoiehLNr7psvCtg/view?usp=sharing

Studies indicate positive benefits to growth in student achievement when teachers use VMs (Moyer-Packenham et al., 2008; Moyer-Packenham et al., 2014; Moyer-Packenham & Westenkow, 2013), but minimal studies have investigated the aspects of PD that support teachers' implementation efforts and transition towards teaching *with* VMs. This article reports the findings related to the aspects of a PD that supported teachers as they prepared VMs to implement with their students. The article concludes with suggestions for supporting and investigating teachers' implementation of technology tools and tasks.

Review of the Literature

Recent studies related to VMs and PD focus on how teachers use VMs in the classroom (e.g., Moyer-Packenham, et al., 2013) and benefits to student learning (Bouck et al., 2015; Moyer-Packenham et al., 2014). However, these studies did not investigate aspects of the PD opportunities that supported teachers' use of VMs. Therefore, this study investigated aspects of a PD opportunity aimed at supporting middle and high school mathematics teachers' use of VMs that teachers reported supported their integration efforts. The following section begins by reviewing recent literature on VMs to highlight benefits to student learning. The review then focuses on the features of effective PD opportunities to set the stage for this study.

Virtual Manipulatives

Over the past 30 years, teachers' implementation of VMs has grown (Moyer-Packenham et al., 2008). Though originally modeled after physical manipulatives and primarily developed as Java applets, VMs now come in a variety of platforms. Moyer-Packenham and Bolyard's (2016) revised definition stated that a VM is "an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge" (p. 13). According to Moyer-Packenham and Bolyard, five most common VM environments exist: single-representation, multiple-representation, tutorial, gaming, and simulation.

Due to the ability to off-load calculations or drawings to some VMs, thus enabling students to focus on content, VMs are a technology tool that can be used to address issues of inequity amongst students and make higher levels of mathematics more accessible to all students (Dunham & Hennessy, 2008). The interactive (sometimes game-like) environment and potential to receive immediate feedback promotes student exploration and perseverance for students who disengage or get frustrated by paper and pencil investigations (Moyer-Packenham & Westenskow, 2013). Additionally, studies suggest that VMs provide equal access for students to learn content by reducing effects of students' demographics (e.g., socioeconomic and English learner status) as predictors of achievement

(Moyer-Packenham et al., 2014). Furthermore, researchers found VMs can be used as a tool for differentiating instruction (Bouck et al., 2015).

Moyer-Packenham and Westenskow (2013) suggested students' mathematical learning is promoted through the five categories of VM affordances: motivation, simultaneous linking, efficient precision, focused constraint, and creative variation. Like any tool, instructional enhancement occurs not due to the properties of the technology itself, but the ways the technology is used within the classroom by the teacher and students and the knowledge and meanings that students develop through their interaction with the technology (Meira, 1998). However, teachers often rely on their own experiences with technology to decide what supports students need to use a tool (e.g., whether they need step-by-step instructions; Zbiek & Hollebrands, 2008). Additionally, studies indicate that student achievement is related to teachers' experiences using the tools (e.g., Moyer-Packenham et al., 2013).

Although finding VMs is relatively easy (e.g., an internet search), VMs may not meet the needs of students in a classroom or support students' developing understanding. Therefore, teachers must critique the instructional value (Suh, 2016) of a particular VM for the ways in which it may support students' emerging understanding of mathematical ideas (Moyer-Packenham & Bolyard, 2016). According to Ladel and Koretenkamp (2016), a need exists for an instrument that helps teachers and designers as they analyze and design VMs that support students' developing ideas.

Professional Development

PD opportunities are an essential feature of efforts aimed at improving student learning (Guskey, 2002). Technology PD related to mathematics education builds from the research related to general PD (Driskell et al., 2016). Therefore, technology-focused mathematics education PD typically incorporates features determined effective in general PD opportunities. Garet et al. (2001) was the first large-scale empirical study investigating what makes mathematics and science PD effective by investigating the relationship between features of PD opportunities and teachers' learning. Since then, numerous studies have built from, corroborated, and added to the findings of this seminal study. Garet et al. identified active learning, coherence, collective participation, content focus, duration, and form as features of effective PD. They found that reform activities (e.g., study groups and mentoring/coaching that occur during the day), as opposed to traditional activities (e.g., workshops that occur outside the school day), led to more positive results and influenced the other PD features.

Perhaps due to the influence of form on the other features, Desimone's follow-up work (2009, 2011) investigating effective PD opportunities focused on the other five features of effective PD (she did not focus on form of the PD opportunity). The following section discusses these features of effective PD opportunities in more detail due to their role in the design of the PD in this study.

Active Learning

Active learning in PD takes on different forms. For example, active learning may provide opportunities for teachers to directly interact with VMs (Albion et al., 2015) as well as support opportunities for teachers to critique and revise VMs. Penuel et al. (2007) suggested that PD might be more effective in supporting new practices when it provides time for discussion, instructional planning, and consideration of fundamental principles of curriculum. Active learning also includes opportunities to discuss potential modifications of a VM and how the modifications may influence student learning related to the teacher's learning goal.

Coherence

Desimone (2009) defined coherence as relating to the alignment of the goals of the PD with teachers' knowledge and beliefs as well as school, district, and state initiatives. Penuel et al. (2007) claimed that coherence is important because teachers filter what they learn from PD and the ways they integrate what they learn from PD through their own interpretative frames.

Collective Participation

Another component of effective PD is collective participation of a group of teachers from the same grade, school, department (Desimone, 2009). Collective participation provides opportunities for teachers to collaborate with each other, discuss how VMs may be revised and implemented, and reflect on current lessons, all of which can be powerful supports for teacher learning (Wells, 2007).

Content Focus

Desimone (2009) contended that content focus may be the most important feature of effective PD, stating that much evidence (see Garet et al., 2001) links "activities that focus on subject matter content and how students learn that content with increases in teacher knowledge and skills, improvements in practice, and, to a more limited extent, increases in student achievement" (p. 184). Therefore, beyond focusing on VMs or mathematics content separately, effective PD provides opportunities for teachers to work with VMs specifically related to their curriculum; thus, promoting teachers in considering how a VM may be integrated and how their instruction and opportunities for student learning may be transformed. Additionally, Martin et al. (2010) suggested that providing opportunities for teachers to work on specific instructional units during PD (where they can receive additional supports) may be more beneficial than focusing on using technology for its own sake.

Duration

Although Yoon et al. (2007, as cited in Martin et al., 2010) suggested a minimum of 14 hours, Desimone (2009) suggested a minimum of 20 hours for a PD. Additionally, the PD should be spread out rather than condensed (Driskell et al., 2016; Garet et al., 2001; Mouza, 2009; Wells, 2007) to provide opportunities for teachers to engage in active learning and reflect on their instruction and possible transformations, as well as

observe others and be observed. Desimone (2009) noted some indication that PD occurring over a semester or possibly an intense summer opportunity with follow-up occurring the next semester is more effective than shorter implementations. A prolonged duration of PD provides opportunities for teachers to see improvements in student learning. Seeing positive effects in student learning is essential for changes in teachers' instructional practices to endure (Guskey, 2002).

Reform Approach

In PD, a "reform approach" means the PD focuses on the context of the teachers involved and provides opportunities for teachers to be immersed in and reflective about the technology tool involved (Garet et al., 2001). Research suggests that PD opportunities that have a reform approach, as opposed to a traditional approach (i.e., short PD opportunity focused on a specific technology tool — often with no connection to teachers' context — and offered over a short span of time), are more effective (Desimone, 2009; Driskell et al., 2016). Wells (2007) claimed that PD designed and implemented from the traditional approach "results in learners who are not well versed on the new innovation at the conclusion of the PD event, and therefore unlikely to realize any lasting change in their practice" (p. 104). Penuel et al. (2007) suggested that reform-oriented PD opportunities have a focus on proximity to practice, supporting teachers' efforts in translating practices from the PD to their classroom practices.

Additional Features

Some studies have advocated for additional features beyond the six identified by Garet et al. (2001). For example, Mouza (2009) suggested that effective PD supports teachers in continually reflecting on their practice and ways they are integrating technology to enhance student learning. Walker et al. (2012) suggested that effective technology-related PD opportunities should have follow-up opportunities that continue to support teachers' efforts in teaching with technology after the PD has ended.

To support teacher learning beyond the PD, Mouza said that PD needs to

engage teachers in planning and implementing *rigorous* technology-enhanced activities in their classrooms. Enactment of new activities and resulting student outcomes can help teachers gather concrete evidence on the importance of technology and foster reconsideration of beliefs. (p. 1237)

Over the past several decades, minimal studies investigating PD aimed at promoting teachers' use of VMs have been published (Driskell et al., 2016). Current studies typically focus on how or why teachers use VMs, or the effects on student learning when they engage with VMs. Furthermore, these studies have primarily focused on elementary teachers.

The study reported here investigated aspects of a PD opportunity for secondary mathematics teachers aimed at promoting teachers' use of VMs as a means to support students' developing understanding (i.e., to help

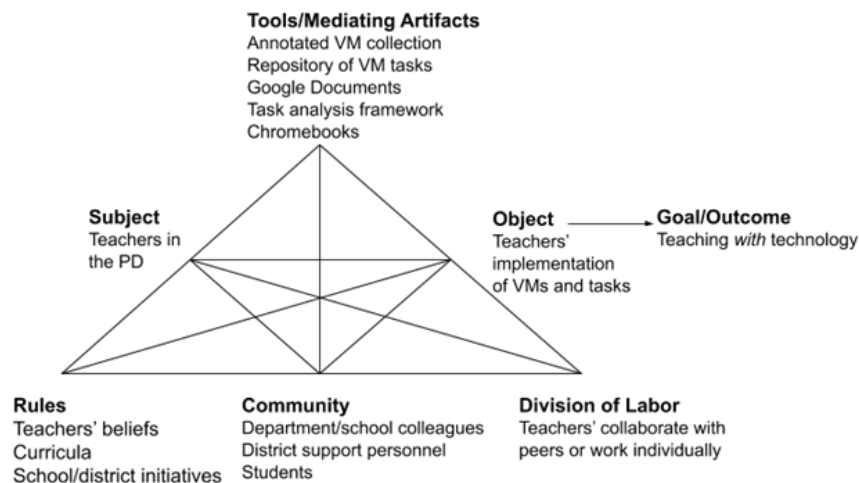
teachers transition from teaching *near* technology to teaching *with* technology). The PD opportunity intentionally integrated features of effective PD opportunities discussed earlier. The following question guided the study: What aspects of a professional development opportunity aimed at promoting teaching with virtual manipulatives supported teachers' instructional practices related to planning and preparing to implement VMs?

Theoretical Grounding: Activity Theory

To investigate the aspects of a PD opportunity that supported teachers' efforts to implement VMs, this study drew from activity theory (e.g., Engeström, 1999). Specifically, this study investigated teachers' participation in the PD (their interactions with each other, the tasks in the PD, their work, etc.), possible transformations to their instructional practices related to implementing VMs, and ways the PD transformed to meet the needs of the teachers involved.

Rather than studying teachers' practices related to preparing to implement VMs in isolation, I investigated teachers' practices in light of mediating factors (e.g., curriculum and school/district initiatives, other teachers and their students, available tools, etc.; Figure 1).

Figure 1 Structure of an Activity System for this PD (Adapted From Reiten, 2020)



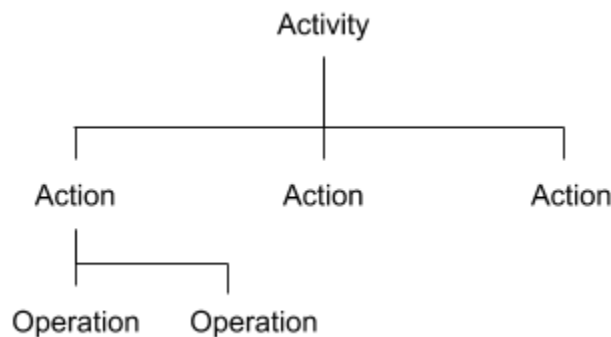
An activity, which is the unit of analysis, consists of a subject, object, and chains of actions, all of which form the context of the activity. In this study, the activity consisted of preparing VMs and tasks to implement during instruction. The *subject* (the teachers in the PD), was the person or people engaged in the activity and was also the learners.

Actions, meaning teachers' instructional practices related to planning and preparing to implement VMs, were goal-directed processes of the subject as they achieved the object. Actions included teachers' practices for selecting, critiquing, and developing a VM or an accompanying

instructional guide. Due to the dynamic nature of the activity system, over time actions may have become operations, meaning that over time, teachers' practices related to planning and preparing to implement VMs may have become routinized and subconscious.

Figure 2 depicts the hierarchical aspect of an activity, actions, and operations. Though conscious, actions are less conscious than activities (Jonassen, 2002). The *objects* are the products produced by the subject. In this study the object refers to teachers' self-reported classroom implementations of VMs during instruction.

Figure 2 Hierarchical Aspect of an Activity, Actions, and Operations of an Activity System



Other aspects of this activity system include the *tools/mediating artifacts* (VMs and the task analysis framework; [Appendix A](#); Reiten, 2018)], *rules* (curriculum and teachers' beliefs about pedagogy), *community* (other teachers in the school and students), and *division of labor* (the ways teachers work individually or in collaboration with others as they prepare to implement VMs and tasks).

This study focused on the aspects of the PD that support teachers' *actions* related to implementing VMs with their students (the *object*). Meaning, this study specifically focused on aspects of the context of the activity system. However, to gain insight into the *actions*, the other components of the activity system must be considered in relation to the aspects of the PD that supported teachers' in preparing to implement VMs and tasks.

Methods

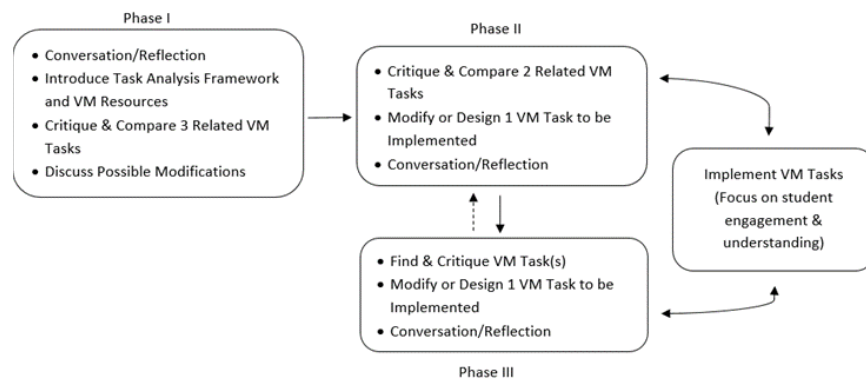
This study aimed to identify aspects of a PD opportunity that supported teachers in beginning to implement VMs and tasks during the course of a PD. The PD opportunity intentionally integrated features of effective PDs. Teachers' developed/modified VM tasks, conversations, responses on Google Docs, interviews, and survey responses were used to investigate aspects of the PD that supported teachers in preparing to implement VMs. The purpose of the PD was not only to introduce teachers to VMs, but to provide teachers with opportunities to interact with VMs and begin thinking about ways to incorporate VMs into their instruction to meet the

needs of their students and promote students' developing understanding (teaching *with* VMs).

Professional Development Design

To support secondary mathematics teachers' implementation of VMs, the PD opportunity implemented a three-phase approach (Figure 3) that supported the expansive learning cycle (Engeström, 1999). The design of this three-phase reform-approach PD was based on features of effective PD. Meaning, the PD focused on teachers' specific mathematics courses (content focus), provided opportunities for teachers to interact with VMs aligned to their upcoming learning goals (active learning), and included 20 contact hours spread across seven months (duration). The PD was coherent with the district initiatives regarding teacher and student technology use and promoted collaboration amongst participants teaching the same courses (collective participation).

Figure 3 Three-Phase PD Model Promoting Teaching With VMs



In Phase I of the model, teachers participated in a whole group discussion about the role of technology in their classrooms and their current technology use. Guiding questions and a task analysis framework ([Appendix A](#)) were tools developed for the PD (see Reiten, 2018, for more information) and introduced to support teachers' actions related to implementing VMs (the object). Revisions to the task analysis framework occurred throughout the PD based on teachers' suggestions for how the tool might better support them in preparing VMs and tasks for implementation.

Phase I constituted the first 4 hours of the PD. Phases II and III of the PD focused on teachers' learning goals and refining their actions related to implementing VMs. In Phase II, teachers were given VMs to critique (see example shareable online document in [Appendix B](#)); whereas in Phase III, teachers selected the tasks to critique and modify using a repository of VMs and tasks curated during the PD. Teachers progressed from Phase II to Phase III at different times due to the attention given to individual teacher's growth.

Though all teachers began Phase II at the same time, teachers did not progress to Phase III until they felt comfortable selecting their own VMs

to explore. As teachers progressed from Phase II to Phase III, they took on the responsibility for finding and critiquing tasks connected to their forthcoming curricula units. This progression aimed to promote the connection between the PD and teachers' practices (as in Wilson, 2008).

The PD met once a month, October through May, for a total of eight sessions. October and November sessions occurred during the school day as two 4-hour sessions held in the morning, and the district provided substitute teachers. To accommodate teacher preferences for meeting, the district math coordinator scheduled the remaining PD sessions as six 2-hour sessions held after school. All PD sessions occurred at the district administration building.

To support teachers' implementations of VMs after the conclusion of the PD (Mouza, 2009), subgoals of the PD included fostering teachers' practices (their actions) for (a) finding VMs and tasks and (b) critiquing and modifying instructional tasks based on their learning goals and the needs of their students. Documents created in Google Docs, a shareable online document platform (see example in [Appendix B](#)), were used during the PD sessions to support individual teachers and the expansive learning cycle. The online documents contained three sections aimed at supporting teachers' practices and tool use. The first section included specific VMs for the teachers to explore (Phases I and II of the PD) or links to the VM repository (located at <http://bit.ly/VMTasks>; see screenshot in Figure 4) and annotated VM resource list (Phase III of the PD). The VM resource list can be viewed at <http://bit.ly/VirtManips> (see screenshot in Figure 5).

Figure 4 Screenshot of the VM and Task Repository

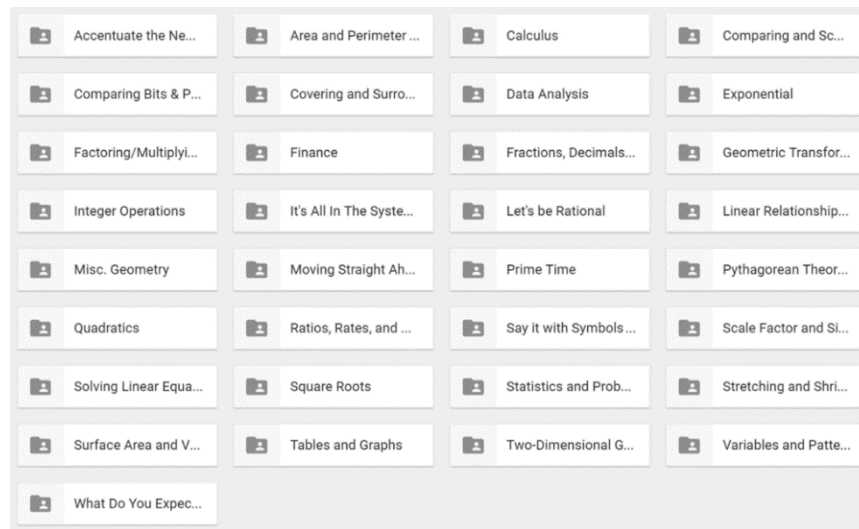
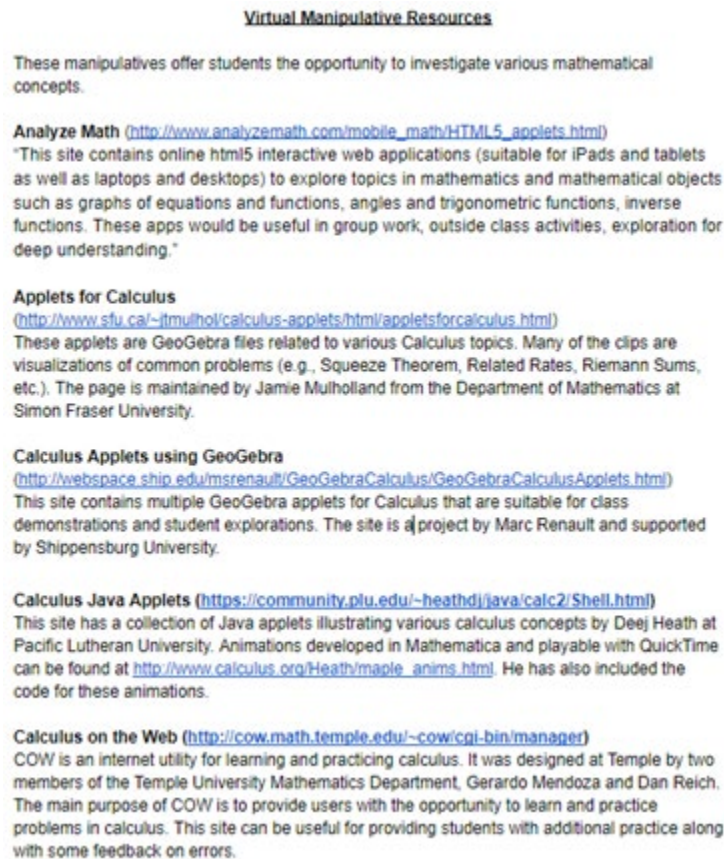


Figure 5 Screenshot of the Beginning of the Annotated List of VM Collections



The repository of VMs and tasks grew throughout (and after) the PD. By the end of the PD, the repository contained over 25 folders with each folder containing five to 30+ VMs and tasks with implementation notes. Likewise, the annotated list of VM collections grew throughout the PD.

In the second section of the online document, teachers responded to guiding questions encouraging them to think about their learning goal and student needs as they critiqued the VM and began thinking about possible modifications. Finally, the third section encouraged teachers to apply the task analysis framework to the VM they were exploring.

Teachers in this study typically developed instructional guides to accompany single-representation environments, thus developing a VM task. A VM task refers to a VM and all accompanying instructional materials (e.g., prompts and directions) whether onscreen or in printed form. Drawing from Sinclair (2003), a VM task may include more than one task focused on investigating a particular concept (e.g., through alternative exploration paths), but it may include only one task. Note, this description of a VM task overlaps with the tutorial VM environment described by Moyer-Packenham and Bolyard (2016), as the VM tasks often guided students as they engaged in mathematical processes and procedures. [Appendix C](#) contains an instructional guide developed by two calculus

teachers during the second PD session to accompany a single-representation VM. The instructional guide combined with the VM formed the VM task.

Researcher Description

I served as the PD facilitator. At the time of the PD, I was completing a doctorate degree in Curriculum and Instruction (mathematics education) at a nearby university. Previously, I taught mathematics for 7 years in middle and high schools and two years at a university. As a mathematics teacher, I was interested in integrating technology-based tasks that engaged students in exploring mathematical ideas and participated in PD opportunities to support this interest. However, at times my integration efforts were not as successful as intended due to lack of resources identifying high-quality technology tasks as well as modifying tasks retroactively rather than proactively (see Reiten, 2018). While completing my doctorate degree, I supported preservice teachers in the teacher preparation program.

Participants

Fourteen teachers from a suburban district in the Midwest participated in the PD. Teachers taught fifth grade through AP Calculus and had 2-20 years of teaching experience (mean: 12.86 years, median: 13.5 years). They represented five schools in the district (see Reiten, 2020 for more details).

Ten teachers, three of whom were intervention teachers, taught Grades 6-8 in middle schools with a one-to-one program, whereby each student was assigned a Chromebook. (In this district, all Grade 6-9 students had their own Chromebooks assigned by the district for the 2015/2016 school year. All teachers in the district were assigned Chromebooks for the 2015/2016 school year. For the 2016/2017 school year, all Grade 5-12 students were assigned their own Chromebooks.) The fifth-grade teacher, who asked to participate in the PD, had an interactive whiteboard and some tablets available in her classroom. One high school teacher had a classroom set of Chromebooks, the other two high school teachers had access to a class set of Chromebooks and a computer lab.

Partway through the PD, the fifth-grade and high school teachers found out that their students would be receiving their own Chromebooks the following school year. Besides the fifth-grade teacher, all teachers taught only mathematics.

Two local districts were approached about the PD opportunity due to their location. I did not know that either district had a recent interest in technology integration within their middle school mathematics classrooms. One of the districts asked to host the PD because district administrators (i.e., mathematics coordinator, technology coordinator, and director of curriculum and innovation) identified that the middle school mathematics teachers typically were not supporting students to use their Chromebooks during instruction.

The math coordinator and I met with teams of mathematics teachers (grouped by grade bands) from each of the middle schools. Teachers were told about the PD opportunity and encouraged to participate, though not required. The math coordinator recruited the teachers from the high school. I did not know any of the teachers before the meetings.

Teachers who chose to participate in the PD earned PD hours that contributed toward their requirement of 50 PD hours per year. Hours were earned based on attendance at the PD sessions, not for work done outside the sessions. Per Institutional Review Board requirements from my institution, teachers had to consent to video and audio recording of the PD sessions to participate in the PD. Attendance at the PD sessions was not required and teachers were able to end their participation in the PD at any time without consequences.

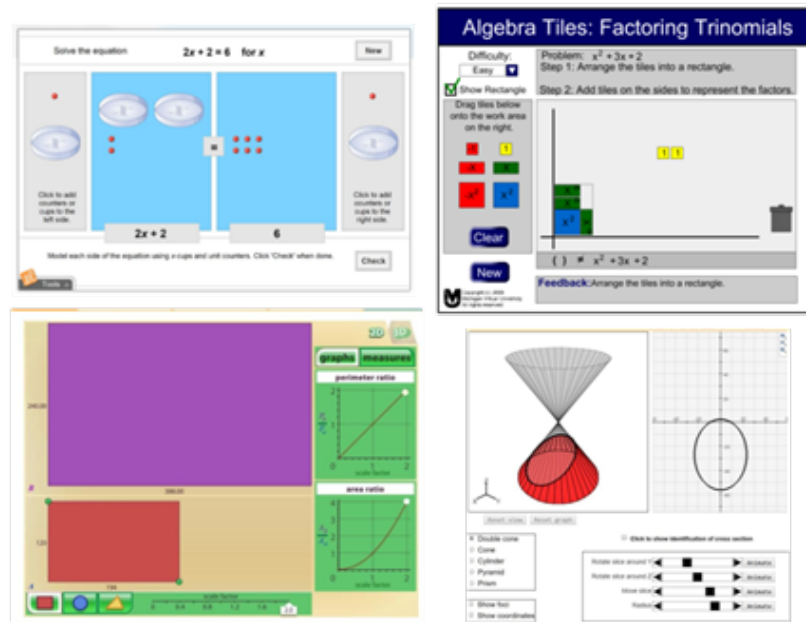
Data Sources

Data collection consisted of an online background survey, audio- and video-recordings of the eight PD sessions, teachers' work during the sessions (written and digital), and an online survey after the PD ended. The initial background survey elicited information from the teachers on topics ranging from number of years teaching to preferred and actual instructional style to their knowledge and experience with different technology tools. Many survey questions were developed based on questions used in previous studies (e.g., Hernandez-Ramos, 2005; Philipp et al., 2003; Wozeny et al., 2006).

The final survey asked teachers about changes to their instructional practices, what supported changes to their practice (if changes were made), what prevented them from making changes, and comfortability and experience with various technology tools. Teachers' work consisted of their responses on the online documents as well as the VM tasks they modified or developed during the PD sessions.

Semistructured interviews were conducted with four teacher volunteers (three middle and one high school) at the beginning and end of the PD. The interviews extended the online surveys by asking teachers to elaborate on changes to their instructional practices, implementations of technology tools, how the PD supported changes in their practices, and challenges that presented themselves during the course of the PD. All recordings were transcribed. Screenshots of the VMs (Figure 6) that the teachers worked with were included in the transcripts of the PD sessions. Teachers' conversations and work during the PD, along with their responses during the final interview and survey, were used to investigate aspects of the PD that supported teachers' actions.

Figure 6 Images of VM Examples Explored and Used by the Teachers During the PD



Data Analysis

Using the constant comparative method (Glaser & Strauss, 1967), transcripts and teachers' work were coded using NVivo. This method was chosen because it focused attention on teachers' experiences and supports through simultaneous coding and analysis. Because components of the activity system mediated teachers' efforts to implement VMs and tasks with their students (object), open coding (Saldana, 2013) was initially applied to the transcripts and teachers' work to identify what supported teachers' actions (i.e., selecting and preparing VMs to use with their students). These themes were then sorted into components of the activity system. Examples of teacher actions included comparing and critiquing tasks, developing/modifying instructional guides, and using a task analysis framework.

After no new codes were identified in the data, recoding of the entire dataset occurred. Additional rounds of analysis ensued to refine code definitions based on commonalities of data within categories. Focused coding then proceeded to identify overall themes in what supported teachers' actions. Connections in the data between components of the activity system (e.g., tools, rules, and division of labor) gave insight into aspects of the PD that supported teachers actions.

All data sources were coded using the same codes (e.g., awareness of resources, task analysis framework, components of the activity system, features of effective PD, etc.). Data excerpts consisted of at least a complete

sentence in each data source. To minimize the potential of segmenting the data too much, at times a datum excerpt contained multiple sentences if the sentences related to the same code.

The findings related to the aspects of the PD that supported teachers' actions were based on responses from 10 out of 14 teachers as four teachers (the fifth-grade teacher, one middle school intervention teacher, and two eighth-grade teachers) did not attend the final PD session, partake in final interviews, or complete the final survey.

Findings

According to the initial background survey responses, teachers had limited knowledge of the variety of VMs that existed. Of the original 14 teachers, three teachers stated that they felt comfortable integrating VMs into their instruction. Six teachers stated they needed additional support to integrate VMs, and four stated that they had not used VMs but wanted to learn more. At the conclusion of the PD, for all 10 teachers, VMs became an instructional technology tool they claimed to regularly integrate in their instruction and felt comfortable doing so. To understand aspects of the PD that supported teachers' knowledge growth related to implementing VMs, teachers' final surveys/interviews, conversations, and work during the PD were analyzed.

The following findings are organized based on themes identified from the final interviews and surveys as to how the PD supported teachers' efforts preparing to implement VM tasks with their students. The first support theme, tools, is broken into two subcategories (task analysis framework and repository of resources). Time was identified as the second support theme by the teachers, and it, too, has two components (active learning and supported planning). These aspects focus on the tools, community, and division of labor components of the activity system described in Figure 1. Connections to the components of the activity system as well as features of effective PD are included in parenthesis throughout the findings.

These findings must be viewed in light of the rule component of the activity system. District administrators, as well as parents and students, expected teachers to integrate opportunities for students to learn with technology in their instruction. This expectation primed teachers to use VMs, not for demonstration but for more inquiry-oriented tasks, whereby students used technology (primarily Chromebooks) to interact with the mathematical ideas rather than only viewing the dynamic representations of the ideas.

Tools

During an activity, subjects (subsequently referred to as the teachers who participated in the PD) transformed tools that are used. In addition, tools mediated how teachers externalized the actions of the object and the mental functioning of the teachers. In this PD the teachers influenced how tools were used, how the tools were modified to better support their use, and how the tools mediated teachers' instructional practices related to preparing to implement VM tasks (i.e., actions). The teachers identified

the repository of VMs and tasks (Figure 4) and the task analysis framework ([Appendix A](#)) specifically as supporting their practices related to preparing to implement VMs and tasks.

Repository of Virtual Manipulatives and Tasks

To promote the use of VMs and tasks beyond the PD (Mouza, 2009), teachers were introduced to an annotated list of VM resources (see <http://bit.ly/VirtManips>) and a repository of VMs and tasks (see <http://bit.ly/VMTasks>) during the first PD session. Nine of the 10 teachers identified that the repository of resources supported their efforts related to selecting and preparing VMs and tasks to implement with their students. The VMs and tasks, which related to teachers' requested content topics, were organized in a Google Drive online folder that was shared with all the teachers (Figure 4). The collections and tasks grew as the PD progressed. The repository provided teachers with a starting point to help focus their search efforts as they became more aware of various VMs and tasks readily available online. Teacher feedback and requests influenced the folder organization, types of included VMs and tasks, and additional support resources (e.g., notes about implementation and instructional guides).

Two reasons developed related to why teachers stated that the PD supported their efforts to implement VMs and tasks regarding the repository of resources. Data indicated that for six of the nine teachers for whom the repository of VMs and tasks supported their actions, the repository did so because it made them more aware of the variety of VMs that existed. For example, during his final interview, Josh (a sixth-grade teacher; all names are pseudonyms) stated, "For me, it's just the exposure to EVERYthing that's out there. That I wouldn't have been able to find a third of it." (Capital letters identify emphasis in speech).

Though the VMs and tasks were readily available on the internet, many of which were free, teachers did not realize the wealth of VMs and tasks that existed prior to the PD. Additionally, five of the nine teachers indicated that the alignment of the VMs and tasks to their specific curricular units or lesson topics (content focus) gave them a place to start rather than searching blindly on the internet. During a final interview with Tracy, (a sixth/seventh-grade intervention teacher and former eighth-grade teacher), she stated,

I think for me, it gave me a starting point. Like I said, everything else was things that I just stumbled upon. As Josh said, it opened our eyes to, "*Holy camole there's a lot out there.*" But now it gave me a place to start from. And now I can TAKE what I've learned and go, "*Ok, this I KNOW this is good, let me check out this one now.*" You know, just gives me a fresh start or a place to start. (Italicized portions identify a change in tone.)

Though a majority of the teachers in the PD found the repository of resources beneficial, three teachers became overwhelmed when selecting a VM or VM task that aligned with their learning goal due to the number

of VMs and tasks in the repository. Jake (an AP calculus, precalculus, and consumer math teacher) stated during his final interview,

It helped when ... I was given one or two, like, “*Here, look at these.*” It was hard toward the end of the year to try and figure out what do, I want to look at, because you just, there was so much information. That I was kind of like, *Which one do I want to look at? And I want to look at ALL of them to kinda see which one do, I want to explore deeper.* And sometimes I spent too much time looking at ... which one I’d want to look at.

By the end of the PD, the repository contained over 25 folders with five to over 30 VMs and tasks per folder. Most folders contained at least 10 VMs and tasks. Jake’s comment identified that rather than the repository of resources helping to focus his efforts, due to the wealth of resources, he struggled to focus his searching efforts. Instead, Jake found himself spending too much time during the PD searching through VMs and tasks rather than using a majority of the PD time developing and modifying tasks to use with his students.

Task Analysis Framework

Given the lack of resources aimed at supporting teaching with VMs, a task analysis framework ([Appendix A](#); Reiten, 2018) was introduced during the first PD session. The framework intended to support teachers as they (a) critiqued VMs and tasks (e.g., to determine the potential for the VM or task to support students’ development of conceptual understanding or to compare VMs and tasks) and (b) modified VM tasks for use with their students that promoted the development of understanding. I did not initially know whether teachers would find the framework helpful. However, eight teachers identified the task analysis framework as a tool supporting their efforts to select and develop VM tasks (actions) to implement with their students. During the course of the PD, affordances and descriptions in the framework were refined based on feedback from the teachers. The revisions intended to clarify distinctions between affordances as well as the ways teachers interpreted the affordances.

The task analysis framework supported teachers’ actions related to implementing VMs and tasks for two reasons. Six of the 10 teachers stated that the PD helped them to be able to critique VMs and tasks to determine whether they might support students’ developing understanding or whether they were only a fun thing to try. For example, according to Mark (a sixth-grade teacher), the task analysis framework supported

... how I think about it and how I JUDGE it.... The framework. Just kind of looking at, questioning things based on those questions [in] the framework. It’s not like I have it memorized, but we used it enough. And so as I look at a different tools that might be something we would use, I think I can, I guess make a good judgement, a better judgement, as to whether this is going to be something that is beneficial to the class or not.

Four of the teachers indicated that the PD helped them to be more thoughtful in how they planned to use VMs and tasks. They began intentionally to modify and develop instructional guides proactively, rather than retroactively. The intent of these modifications was so that students' engagement with the VMs and tasks focused on the learning goal rather than irrelevant challenges (e.g., poorly worded prompts, investigations that lacked a goal, etc.). Jake's response represents the change in how his and other teachers' actions changed regarding implementing VMs and tasks with their students. Jake stated,

I think that the PD helped me kind of think through some things. As opposed to just thinking, *Ahh well, we'll try it and see what happens, and then we'll kind of modify afterwards ...* being a little more thoughtful about how this is going to be used and what type of questions should be asked. Or do I need to modify this ... worksheet that goes along with this, so that it's going to help beforehand as opposed to, *Oh, well that didn't go the way I really wanted it to go.* And then you're doing it after the fact.

Critique VMs. One subgoal of the PD was to support teachers as they critiqued VMs and tasks that they found online. During the initial PD session, several teachers commented that they often implemented tasks they found online because they “looked fun” (a misuse of technology, Suh, 2016) rather than focusing on how the task might support students' development of understanding. However, during the course of the PD, teachers identified the task analysis framework as supporting their efforts to critically evaluate VMs and tasks to determine whether the task might be beneficial for their students (part of the community component of the activity system) or only a fun thing to try. For example, during the March PD session, Daron (a geometry and AP statistics teacher) said,

I am looking at the framework and seeing what I want to change based on that. *I think the framework does help you focus on like, different levels ... to activate that background knowledge and get them thinking about what they already know. And then trying to, push them, push that forward too.*

According to Tracy, the framework helped her and her colleagues to “look at — CRITIQUE them more critically and look for their VALUE versus just, a fun thing to try.”

Modify/develop instructional guides. During the first PD session, teachers engaged with one VM that had an instructional guide and one single representation environment VM. After this experience teachers quickly gravitated toward VMs that had some type of an instructional guide or minimally had accompanying exploration questions. During this first PD session, teachers commented that instructional guides were helpful because they were more guided, gave students something to refer back to, and helped to keep students focused on the learning goal rather than “playing” (i.e., clicking through without making any connections).

For example, Tracy used a gizmo from ExploreLearning during the prior year with her eighth-grade students, but she did not realize that gizmos had accompanying instructional guides. Upon learning about the

instructional guides during the October session, she became vocal with her tablemates and the group at large regarding her thoughts about instructional guides and how beneficial they would have been for her students:

I just think that it is more GUIDED and that makes it a little more REAL to them where they are actually having to do what they would have to do on a math test, you know. Like, that they are practicing those skills and, writing it down and maybe making it a little more concrete than just, click, click, click, click, click, click, click. And, see if we get an answer eventually that says, *Hey, you're right!* [chuckles] So, I just think that it is more guided for them.

Although not an initial goal of the PD, teachers quickly developed the desire (or need) to have instructional guides of some type to accompany VMs. Beginning with the second PD session, all teachers either developed or modified printed instructional guides or implemented VM tasks that had onscreen prompts (i.e., VMs in tutorial or simulation environments). Therefore, as the PD progressed, focused support regarding how to construct or modify instructional guides was given (e.g., potential questions to add, specific instructional guides to look at for guidance, structural modification suggestions, etc.).

Though instructional guides primarily occurred in print form rather than in an onscreen format, Daron created an online Google Form containing questions his students responded to as they interacted with a chi-squared VM created using GeoGebra. During later PD sessions, teachers often asked whether a VM had an accompanying instructional guide before they began exploring it. Of the 10 teachers reported on in this study, six teachers identified the task analysis framework as supporting their process for modifying/developing instructional guides to accompany VMs and tasks. For example, during the January PD session, Daron said,

Some of the ones that I had been looking at don't really have a guide, so that's like [pause], "*Yeah, this,*" [the framework] I kind of keep these things in mind as I try to write a guide or put together some questions. For these, to try to make sure as many of these were covered (i.e., referring to the affordances in the framework) so it was a meaningful activity for them as well. Where otherwise you might just throw the questions out, you just, I think it allows you to think more like, "*How do I move them up to those levels that we're trying to get them to?*"

Daron talked about making an instructional guide or putting together some questions so that the task was meaningful for the students and promoted opportunities for developing understanding. He said that he used the framework to help him write a guide that was intentional and meaningfully moved students to the levels of understanding that he wanted, as opposed to throwing questions out without a focus.

Operationalizing Applying the Task Analysis Framework. In final surveys and interviews teachers more frequently stated that the task analysis framework helped them critique a VM or task rather than helped them to modify or develop an instructional guide. However, during Phase

III PD sessions, teachers more explicitly used the framework to support their efforts in modifying and developing instructional guides as opposed to critiquing VMs and tasks. For some teachers, their use of the framework to critique tasks possibly became operationalized (a subconscious action). For example, during Josh's final interview, he said,

Yeah, the framework thing. I mean, in the beginning, it was like, *Well, yeah, I guess that helps you think about is it worth DOING or not.* And then by the time we did the, you know, the last few, we didn't even, it was already in your HEAD. That, *how-wh-what would I change to make it more worthwhile or is it fine the way that it is?* 'Cause you kind of already have that down. But initially, it was confusing. I thought, I think it helped kind of pick out what was kind of, what was needed.

Josh's reflection highlights how he found the framework challenging to apply in the beginning of the PD (e.g., due to overlapping affordances, not understanding an affordance, etc.). Through framework modifications, conversations with peers, and repeatedly using the framework to critique different VM tasks, it became second nature for him to apply the framework as the PD progressed. Final interviews and surveys indicated that the repository of VMs and tasks, as well as the task analysis framework, were two tools that were instrumental in supporting teachers' actions for implementing VM tasks.

In the subsequent section, the findings turn to focus on the last support theme. Teachers identified time as an important aspect of the PD for supporting their instructional practices and begin moving toward teaching *with* VM tasks.

Time

The PD was specifically designed so that teachers spent most of their time during the PD sessions interacting with VMs and tasks (active learning) related to their instructional units (content focus) and working with their peers (collective participation) to prepare how they were going to implement the VM tasks. This time was built into the PD to provide teachers opportunities for active learning and collaboration, but I did not know whether teachers would find this time useful. Six teachers identified time as a feature of the PD that supported their efforts to implement VM tasks.

Active Learning and Content Focus

Five of the six teachers who stated time supported their efforts identified the active learning and content focus features of effective PD opportunities (i.e., interacting with VMs and tasks directly linked to their instructional units) as supporting their actions related to implementing VMs and tasks with their students. These aspects of the PD strengthened the link between the PD and teachers' practice (Wilson, 2008) and supported teachers in meaningfully integrating VM tasks related to their lessons rather than as an add-on component that was not actually connected to their lesson goal. For example, according to Curt (a seventh-grade teacher), one of his

biggest takeaways from the PD was knowing “what activities align with the CMP3 books.” (CMP3 refers to the third edition of the Connected Mathematics Project from Michigan State. CMP 3 was the curriculum used in grades 6-8.)

Time to Prepare Virtual Manipulatives and Tasks with Support

Five of the six teachers reported that a contributing factor to their implementation efforts was how the PD supported opportunities to prepare VM tasks (actions) to implement with their students. Teachers identified the task analysis framework (discussed previously) as an important tool supporting their efforts to critique and modify or develop instructional guides promoting students’ developing understanding. During the PD, they had time to discuss and use the task analysis framework with their peers and PD facilitator (the author) as they critiqued VMs and VM tasks and as they prepared instructional guides. Teachers’ actions were further supported through having time to collaborate with their teaching peers (i.e., teachers who taught the same grade) and teachers in nearby grades (teachers who taught within one or two grade levels), thus relating to the collective participation feature of effective PD opportunities. For example, during Josh’s final interview, he said,

I liked when we were all there, Tracy, Mark, myself, Karen. When we had a large amount of six-seven people there. Just the chance to bounce ideas off, or someone looking at it from a different perspective and saying, “*Ahh, we could totally use this one for*” you know. Or “*After this lesson.*” And it was like, “*Ah yeah, you could.*” And, where THAT is so valuable to have that—that group time.

Josh’s reflection highlights how the community (other teachers) component of the activity system mediated his actions and resulting implementation of the VM tasks. Mark and Josh were sixth-grade teachers, and Tracy was a sixth/seventh-grade intervention teacher. These three teachers were at the same school. However, Karen was a seventh-grade teacher at a different school in the district. Josh had not worked with Karen before, yet identified Karen specifically as supporting his efforts to implement VMs and tasks. Thus, Josh’s community supported his actions. Additionally, Josh’s reflection highlights the division of labor component of the activity system when he described the opportunity to bounce ideas off of each other or suggest VM tasks to each other thus distributing the responsibility for selecting and preparing VM tasks to use with students.

Continuing Beyond the PD

Teachers’ experiences with VMs and implementing them in their classrooms extended beyond the PD sessions. For example, Mike (an AP calculus, pre-calculus, and transition to college math teacher) shared that he designed a project to use after the AP calculus exam whereby his students would search for “and find a few manipulatives that may have helped them. Then see if they can create their own.” During the following school year, four teachers reached out on their own to say that they were

still using the resources from the PD and implementing additional tasks into their instruction. Three teachers also requested to share the resources with additional teachers at their school (their community), thus promoting the use of VMs and tasks to teachers outside of the PD (as also in Mouza, 2009).

Discussion

This study drew on the central tenets of activity theory by considering teachers' actions (i.e., their instructional practices related to selecting and preparing VMs and tasks) for implementing VM tasks, as mediated by other components of the activity system. By better understanding how the various components of an activity system mediate teachers' actions, teachers can be supported to teach with VM tasks and technology, more generally. Teachers' reported efforts implementing VMs with their students (the object) suggest they began transitioning toward teaching with VMs and tasks (the goal). Classroom observations and interviews with students as well as analysis of student work are needed to know whether teachers were teaching with VMs and tasks. Therefore, this study focused on teachers' actions related to the object and not the goal of the activity system. At the conclusion of the PD, teachers indicated that VMs became one of the technology tools that they regularly integrated into their instruction. Findings indicated that the repository of resources, task analysis framework, and time were key aspects of the PD supporting teachers' efforts preparing to implement VMs and tasks with their students.

Features of Effective Professional Development Opportunities

The PD was intentionally designed from a reform approach (Driskell et al., 2016; Penuel et al., 2007) and implemented features of effective PD opportunities. Throughout the 8-month PD, teachers' conversations during the PD and interviews provided opportunities for them to reflect on and discuss topics, such as the VMs they were exploring, the ways they implemented or may implement a specific VM task, possible modifications to make to a given VM task to further support students' developing understanding, and ways their students responded to the VM tasks.

The PD provided opportunities for active learning (e.g., Desimone, 2009; Driskell et al., 2016; Martin et al., 2010), whereby teachers directly interacted with VMs and tasks aligned to their upcoming curricula units. Focusing on the upcoming curricula units relates to the content focus component that Desimone (2009) claimed as one of the most important features of effective PD opportunities. Intentionally designing the PD whereby teaching teams (collective participation) joined (as opposed to individual teachers) provided opportunities for teachers to collaborate with each other to plan and prepare ways to implement VM tasks (Mouza, 2009), rather than individual teachers working in isolation.

Aspects of the PD Supporting Teachers' Implementation Efforts

Teachers identified the repository of resources and the task analysis framework as two tools that supported their implementation efforts. Due to the investigative nature of the teachers' curricula, teachers primarily searched for VMs that encouraged student investigation and inquiry rather than drill and practice (relates to rules of the activity system). As the PD progressed and teachers became more versatile in using the repository of resources, they focused their open searches using the repository and the annotated list of VM collections that were the most relevant to their needs (e.g., based on curriculum focus and student needs). The repository of resources became overwhelming for some teachers, however, due to the number of different collections. Teachers in the PD were more productive in using the repository of resources when they had a particular learning goal in mind as opposed to, as one teacher said, "looking at what's out there."

Teachers must be prepared to create high-quality engagement with VMs and tasks that present opportunities for students to develop understanding. Teachers stated that the task analysis framework helped them critique VMs and tasks as well as modify and develop instructional guides proactively rather than retroactively. The task analysis framework served as a *tool* to support teachers in critiquing, modifying, and developing VM tasks, whereby students were interacting with the VM and its characteristics to explore a mathematical idea through communicating, reflecting, and connecting mathematical representations.

Only through judicious use of VM tasks are students supported in developing understanding of mathematical ideas (Moyer-Packenham & Bolyard, 2016). Therefore, teachers must be able to critique the instructional value of any technology tool (Suh, 2016).

The task analysis framework addressed the call by Ladel and Kortenkamp (2016) "for an instrument that helps when analyzing and designing" (p. 29) VMs and VM tasks. Furthermore, critiquing VMs and tasks helped teachers distinguish between mundane and powerful uses of technology (Wilson, 2008).

Though not a tool identified by the teachers as supporting their integration efforts, online shareable documents that teachers responded to during the PD sessions supported their use of the tools by providing individualized support as teachers engaged in the expansive learning cycle (Engeström, 1999). The three-part online documents provided teachers direct links to the annotated list of VM collections as well as the repository of resources. In this manner, the one document approach served as an organizational template that provided teachers with a lens to focus their search efforts (asking them to identify their learning goal in Part I), then select and investigate a VM or VM task (Part II), and then apply the task analysis framework to the VM task before and after modifications were made (Part III). Initially, the online document was individualized for each teacher based on their upcoming curriculum focus. For example, the online documents initially included links to specific VMs for teachers to investigate (Phase I of the PD), then suggested tasks (Phase II of the PD),

and finally, only links to the repository of resources and annotated list of VM collections for teachers in Phase III of the PD. Teachers in Phase III of the PD received the same generic online document.

Though the online document platform was initially intended as a data collection tool and to direct teachers to specific VMs and tasks to explore (thus providing some individual support), teachers became accustomed to responding to the online documents and expected them for each PD session. As the PD progressed, some teachers began working on the online document before the PD sessions (e.g., when they had prep time earlier in the day). Through repeated use of the task analysis framework, teachers not only became more comfortable applying the framework but some teachers operationalized its application. That is, some teachers began applying the framework automatically or subconsciously while they were investigating a VM or VM task. This study did not examine whether teachers would have applied the framework to regularly critique and modify or develop VM tasks had they not been responding to the online documents.

Acknowledging Mediating Components

It is important to acknowledge the role that VMs and tasks themselves had in supporting teachers' implementation of the VM tasks. Teachers quickly bought into using VMs because VMs allowed them to use their Chromebooks more frequently and effectively in their instruction, thus supporting the district initiative (coherence) related to technology use in the classrooms (part of the rule component of the activity system). Because many of the VMs that teachers selected had accompanying guided questions or instructional guides, teachers often modified existing resources rather than designed instructional guides from scratch. Choosing these VMs and tasks decreased the time teachers spent preparing the VM tasks compared to developing the accompanying instructional guide from scratch. Additionally, teachers were drawn to VMs due to the ease of use and the immediate benefits that they saw related to student engagement and understanding. Seeing improvement in student learning is often essential for teachers to continue (Guskey, 2002) implementing VMs tasks.

Investigating teachers' actions (their processes for preparing to implement VM tasks) in conjunction with other mediating factors (district initiatives, tools, community, etc.) gave insight into teacher's implementation efforts. For example, Josh and Mari (an eighth-grade teacher) attended the same number of after-school sessions, taught from the same curriculum series, and talked about using VM tasks not only to supplement their instruction but to differentiate their instruction as well. However, due to the influence of their teaching team members (community), Josh regularly implemented VM tasks whereas Mari may have implemented one VM task during the PD.

Studies indicate positive benefits to growth in student achievement when teachers use VMs (e.g., Moyer-Packenham et al., 2008, 2014; Moyer-Packenham & Westenkow, 2013), yet the frequency of VM use decreases in the middle and high school grades (Moyer-Packenham et al., 2013). Therefore, this study investigated the actions of and reported

implementation efforts of secondary teachers who primarily did not know about or feel comfortable integrating VMs on their own prior to the PD opportunity. Yet, by the end of the PD, all 10 teachers reported implementing VMs regularly and felt comfortable doing so.

As the wealth of VMs increases for secondary mathematics content, the focus of this study sought to better understand how secondary mathematics teachers can be supported to teach *with* VM tasks. After all, studies indicate that student achievement is related to the teacher's experience using the VM (e.g., Moyer-Packenham et al., 2013).

Implications

Drawing from the findings, implications from this study exist for supporting teachers to implement VMs and technology tools, more generally. Aspects of this PD that supported teachers' efforts to implement VM tasks should be considered for future PD opportunities. Important aspects of this PD included providing teachers with tools (e.g., the task analysis framework and repository of VMs) that supported their integration efforts, as well as time to use the technology tools that aligned with their curricular goals and collaborate with teaching peers who might support teachers more generally in teaching *with* (as opposed to *near*) technology tools.

When supporting teachers' use of VMs and tasks, teachers should use their learning goal to concentrate their search efforts rather than trying to match a learning goal to a VM or task that they may find. Until teachers become more knowledgeable about the wealth of resources for selecting tasks on their own, they should focus their attention on a limited number of tasks (e.g., two or three) or specific collections.

This gradual progression of teachers taking on more responsibility for finding the VMs and tasks supports the expansive learning cycle (Engeström, 1999). When supporting teachers in applying tools introduced in a PD, consideration must be given to the ways teachers receive individual support and encouragement to use the tool during the PD (e.g., the online document). Future research is needed to further explore why the teachers found the task analysis framework beneficial in supporting their efforts to implement VM tasks. For example, one line of future research may investigate features of the task analysis framework that supported teachers' actions.

When considering why teachers choose to implement a particular technology tool, consideration must be given to the features of the tool itself and existing resources that may ease teachers in adding the technology tool to their instructional repertoire (e.g., instructional guides, task analysis framework, etc.). Additionally, when investigating aspects of a PD that support teachers' use of a new technology tool and the ways the PD supports their implementation efforts, factors outside the PD (e.g., community, including teachers' students and teaching team members) must be considered that mediate teachers' instructional practices and technology use.

Conclusion

Most studies related to teachers' use of VMs focused primarily on elementary teachers, with few studies focusing on middle and high school teachers (Moyer-Packenham & Westenkow, 2013). As VMs and tasks related to secondary mathematics content grows, a study was needed to investigate aspects of a PD opportunity that supported secondary mathematics teachers' implementation of VM tasks. Rather than one specific aspect of the PD supporting teachers' implementation efforts, teachers' implementation efforts were mediated by various components of their activity system. Middle and high school mathematics teachers implemented VMs and tasks in their classrooms (and felt comfortable doing so) due to tools introduced during the PD (i.e., the task analysis framework and repository of resources). Additionally, having time to interact with VMs and collaborate with their peers to select and prepare VMs tasks for their students further supported their implementation efforts.

Future studies need to explore whether additional aspects of PD opportunities support and prepare middle and high school mathematics teachers to teach with VM tasks. These studies may identify additional tools or support resources needed for preparing teachers to effectively integrate technology to support students' mathematical learning opportunities.

Supporting teachers to teach with technology goes beyond providing access to technology tools. Rather, it includes providing opportunities for teachers to interact with and try out the technology tools integrated within their current curriculum. The findings indicate that when teachers are supported in learning about VMs and ways VMs can be used to promote students' development of understanding (i.e., teaching with technology), teachers began implementing VM tasks and reported feeling comfortable doing so.

Although the findings and outcomes of this study are activity specific (i.e., specific to this PD), the findings inform future efforts aimed at promoting teachers' use of technology-based instructional tasks, whereby teachers teach with as opposed to near technology. During the PD, teachers focused on using VMs due to their potential for promoting understanding, as opposed to being only a fun thing to try because the VMs were cool or because they had to due to external pressures. The tools introduced in the PD, the structure of the three-phase PD model, and many of the suggestions related to supporting teachers' efforts to teach with technology-based tasks would be helpful in promoting preservice teachers' efforts to teach with as opposed to near technology.

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Contemporary Issues in Technology and Teacher Education is an online journal. All text, tables, and figures in the print version of this article are exact representations of the original. However, the original article may also include video and audio files, which can be accessed online at <http://www.citejournal.org>

Appendix A

Task Analysis Framework Given to Teachers

Task Analysis Framework

This framework is intended to help teachers better critique and develop tasks aimed at promoting students' development of conceptual understanding of mathematics through reflection and communication (Hiebert et al., 1997), as well as through using and connecting mathematical representations (NCTM, 2014). Descriptions below are not necessarily in a hierarchical ordering nor are they mutually exclusive.

Portions of the table below are adapted from Trocki (2014) and Sinclair (2003).

Affordances	Descriptions
N/A	Task is primarily a technology task with no focus on mathematics.
N/A	Virtual manipulative does not have mathematical fidelity required to respond to the prompts.
A	Task prompts students to recall a mathematical fact, rule, formula, or definition.
B	Task prompts students to report information from the virtual manipulative or consider mathematical concepts, processes, or relationships in the current display. The student is not expected to provide an explanation.
C	Task provides opportunities for students to explain the mathematical concepts, processes, or relationships in the current display.
D	Task provides opportunities for students to make predictions and then test their predictions using the virtual manipulative.
E	Task provides opportunities for students to connect multiple representations of a mathematical concept (e.g., graphical, algebraic, and tabular representations of a relation).
F	Task provides opportunities to check students' understanding of mathematical concepts, processes, or relationships. Task may provide minimal feedback to the student based on specific errors.
G	Task provides opportunities for students to go beyond the current display by considering multiple examples to generalize mathematical concepts, processes, or relationships.
H	Task supports students' exploration through manipulation of the display that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise (e.g., addressing a common student misconception).

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Heinemann.

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Appendix B

Contents of Example Shareable Online Document

PD Day 7: Modifying a VM Task

Please respond to the prompts below. The prompts are intended to help you modify the VM tasks with your students in mind.

Choose one of the tasks listed in the **VM Tasks** folder. Or, if you have a topic in mind that is not listed, you may use the links on the **VM Resources** page to find a new task.

Which task did you choose to modify? _____

Complete the student exploration handout for your task. Available instructional guides are located in the various folders. If you download a new one, please share it with me. If no handout exists, spend some time exploring the VM and then respond to the questions below.

1. What is an essential question you would use this task to explore?
2. Where do you think your students might struggle in this task?
3. How might your students benefit from engaging in this exploration?
4. Thinking about the essential question, what is one modification you would make so that the exploration better fits the needs of your students? Why might this modification help your students engage in the essential question?

Now, use the framework that we have been talking about (i.e., the **Task Analysis Framework v3**) to help you think about how to modify/develop the task.

1. Which prompts of the framework apply to the activity you chose?
What from the activity supports your claim(s)?

Now, use the framework to help you modify the task to better fit the needs of your students. Please email or share with me the document that you create. You may want to shorten the task or modify some of the prompts to better align with your learning goals and the needs of your students. Or, maybe you will need to create an instructional guide.

After modifying the task, which prompts of the framework apply to the task you chose? **What from the activity supports your claim(s)?**

Appendix C

Instructional Guide Developed by Calculus Teachers During PD Day 2

AP Calculus Graphing Derivatives and Functions

Name _____

You have two websites shared with you on Padlet.

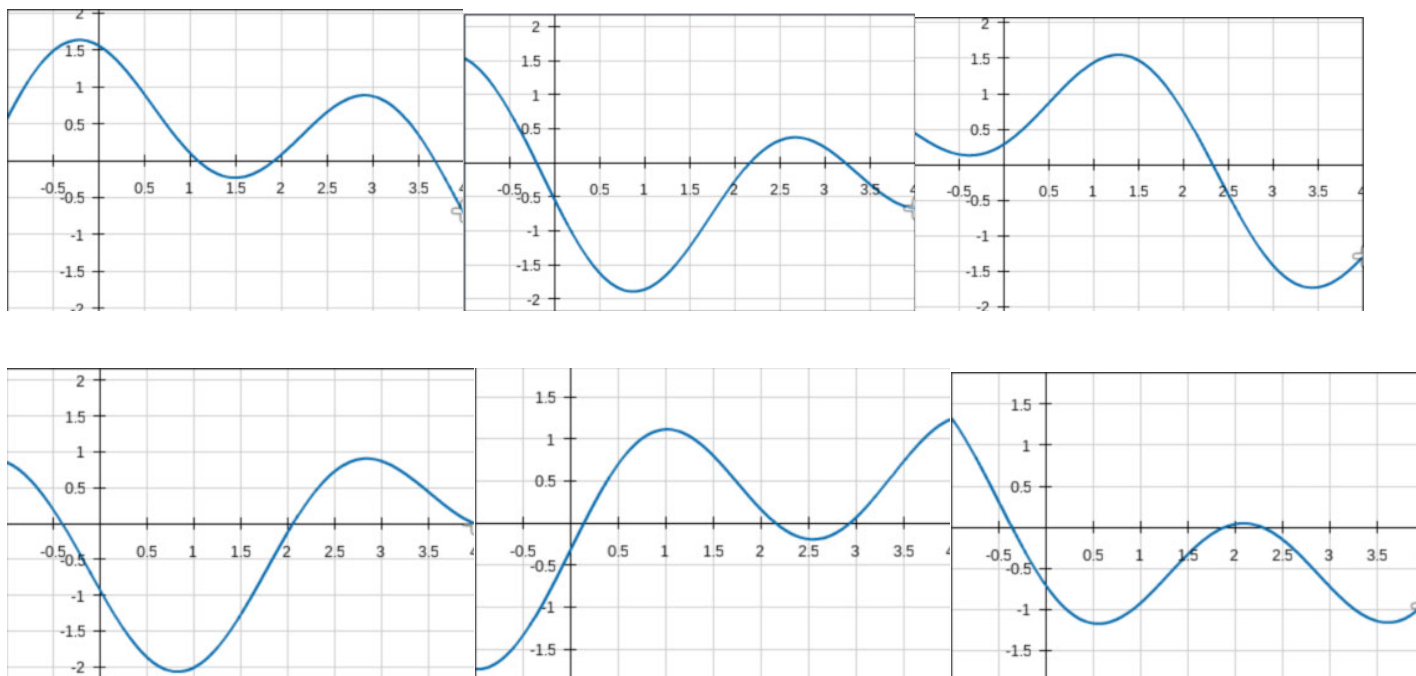
<http://www.flashandmath.com/mathlets/calc/derdraw/DerivativeDraw.html>

I would like you to thoroughly explore both sites. Practice graphing a derivative given a function and practice graphing a function given a derivative.

Using the first site what was the highest score you received? _____

Which types of functions were the most difficult to sketch? _____

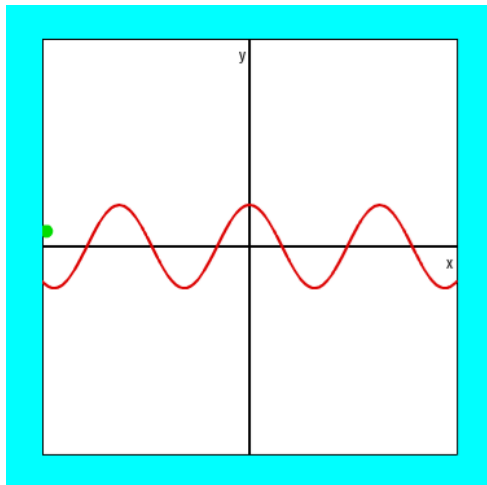
If you were to teach a classmate how to sketch a graph of a derivative given the graph of a function what would be the three most important things to remember?



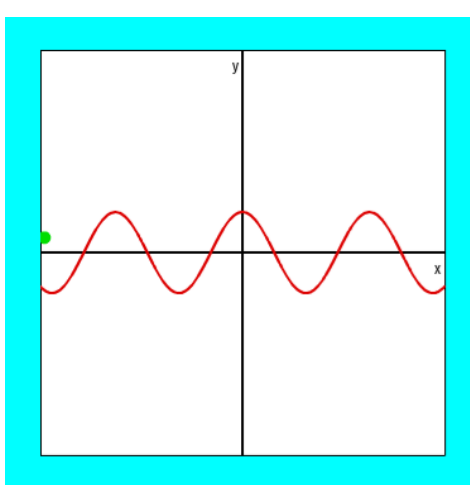
<http://www.ltconline.net/green/java/Other/IntegralGraph/SketchFfromFPrime.>

As you navigate the second website sketch your guess here, then try it on the chromebook, be sure to record your score, keep trying until you get a score of 90 or better.

Prediction

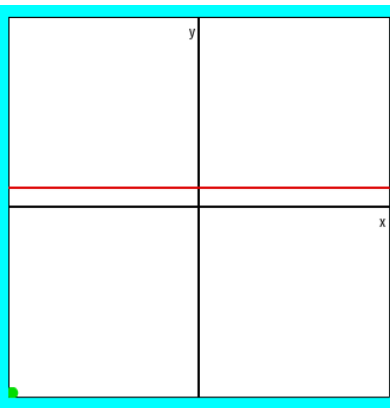
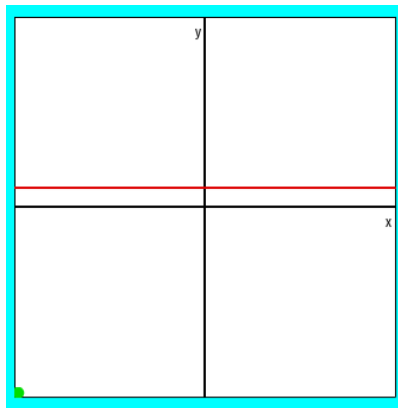


Actual



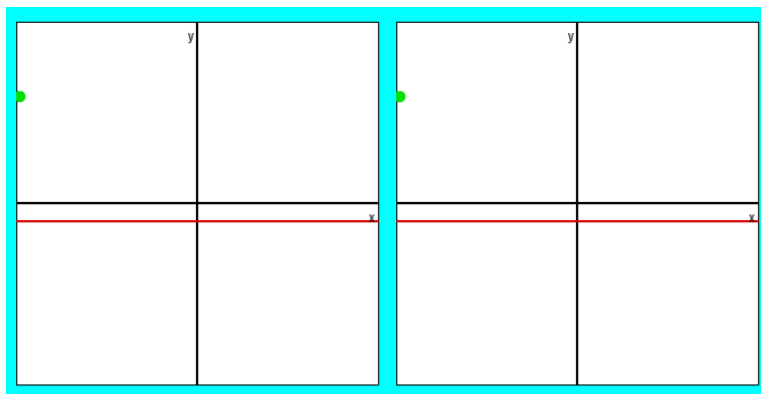
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2nd Score _____



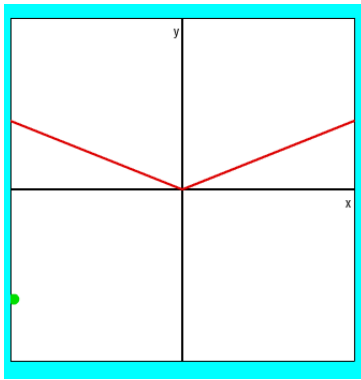
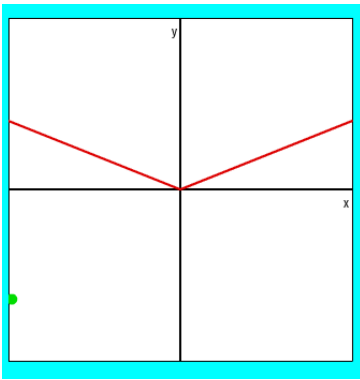
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2nd Score _____

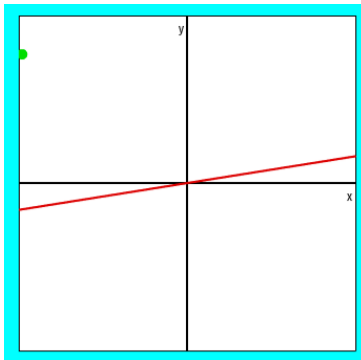
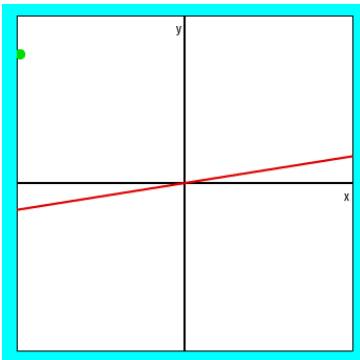


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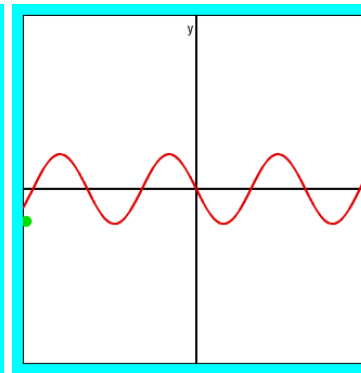
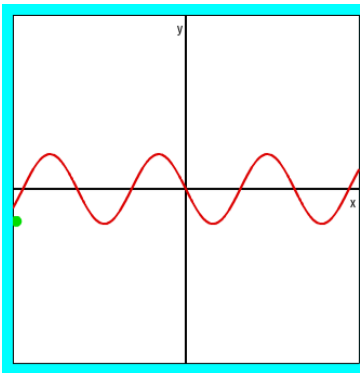
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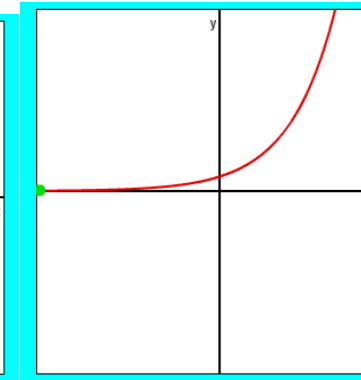
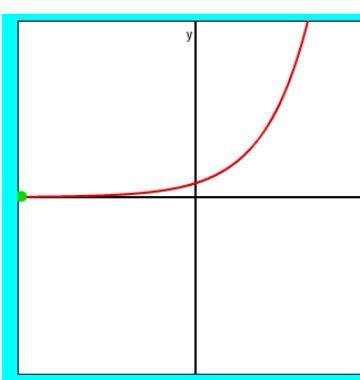
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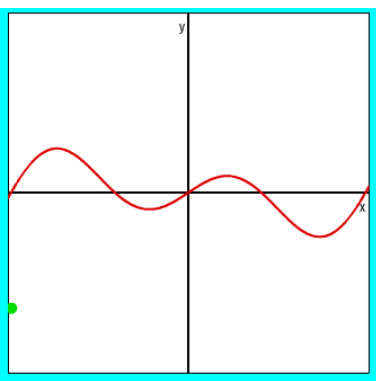
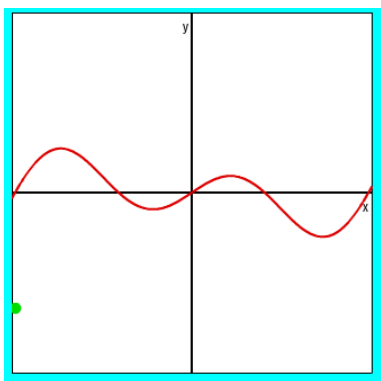
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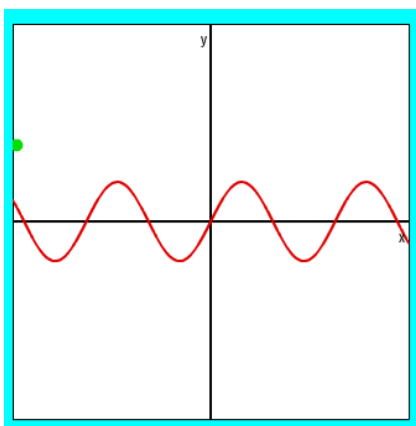
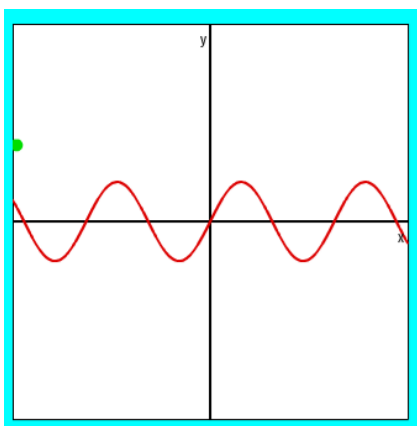
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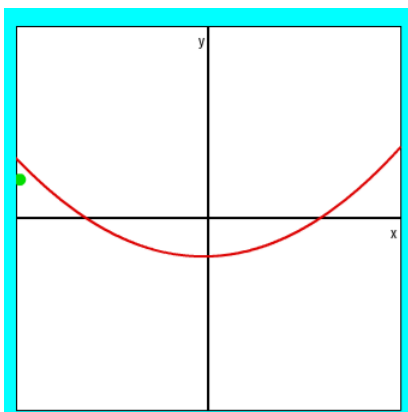
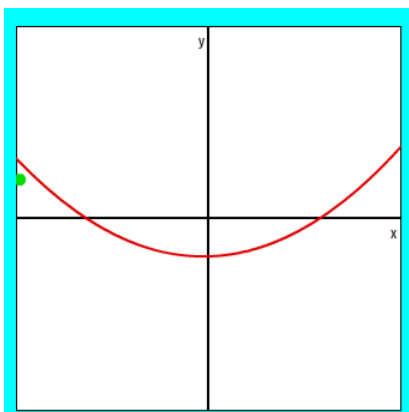
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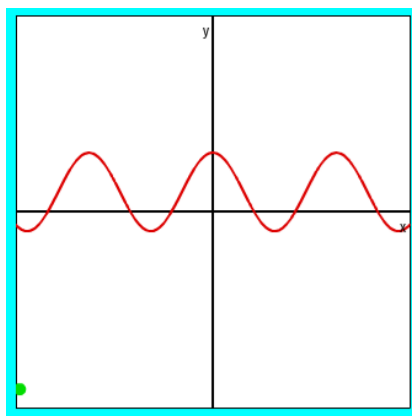
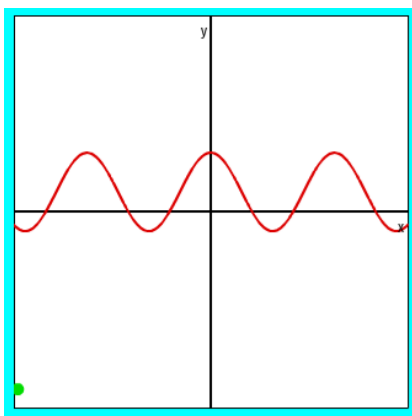
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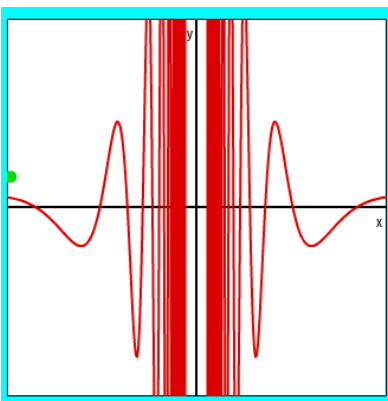
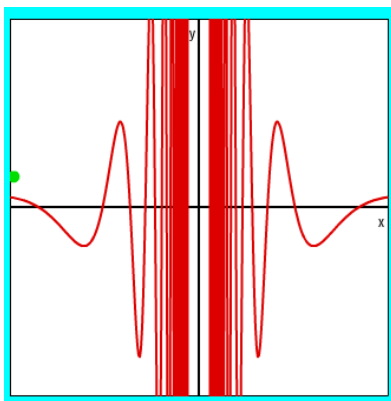
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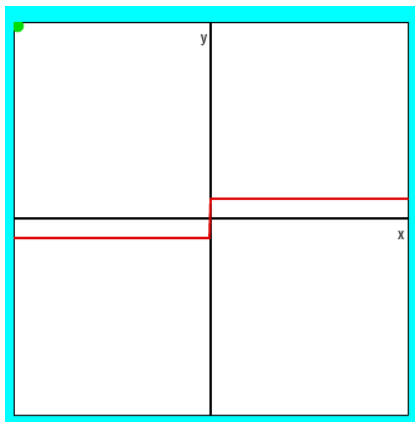
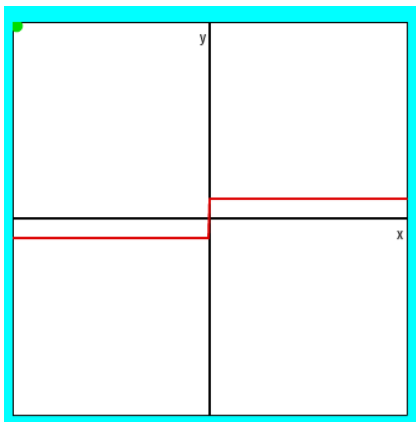
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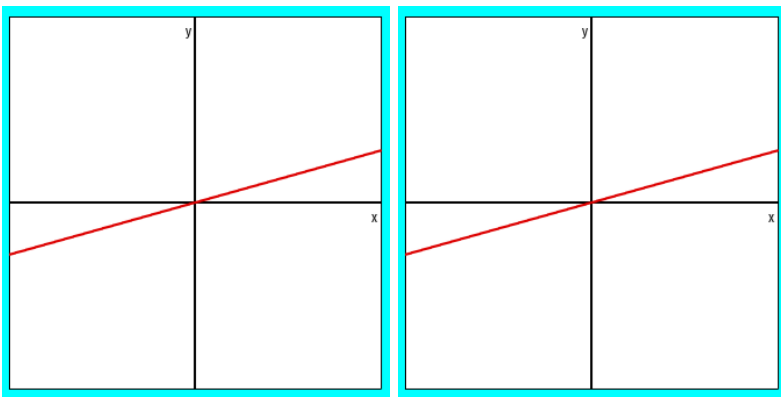
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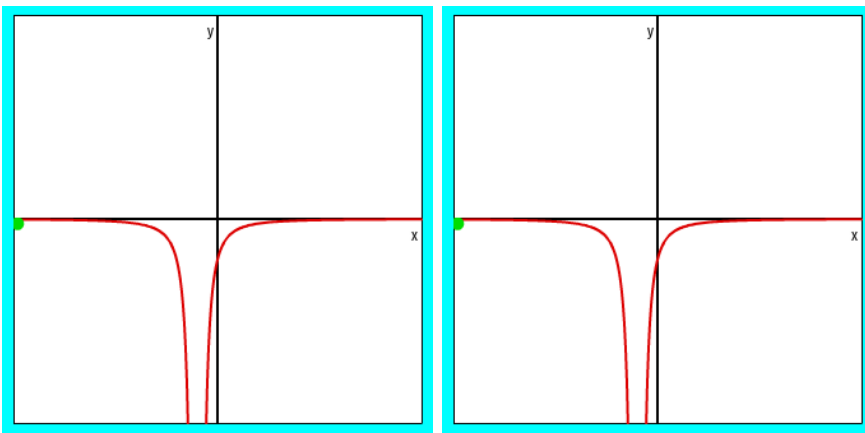
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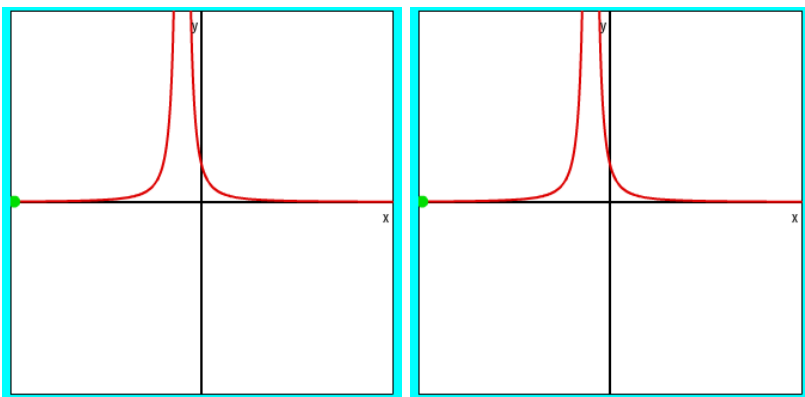
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2nd score _____



1st score _____
2nd score _____

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