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Coding for the Core: Computational Thinking and Middle Grades Mathematics

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National standards and frameworks for mathematics, computer science, and technology emphasize the importance of teaching all children computational thinking (CT) skills. These skills are important for preparing citizens that are literate in science, technology, engineering, and mathematics and for participation in a society that is rapidly changing with emerging technologies. This paper describes a 72-hour summer institute for grades 6-8 middle school mathematics teachers (n = 22) with a comprehensive approach to professional development, including training in computer programming with Bootstrap Algebra and Lego® Mindstorms® robotics, mathematics content sessions, and mathematics pedagogy sessions. Results of an assessment used to measure content knowledge and CT skills as well as the Technological Pedagogical Content Knowledge survey yielded statistically significant increases. Participant reflections revealed they valued opportunities for collaboration within grade-level professional learning communities and integration of CT strategies through both programming and robotics. Based upon participant feedback we recommend choosing either the use of Bootstrap Algebra or Lego Mindstorms within shorter timeframes to better prepare teachers for classroom implementation. These middle school teachers were receptive to mathematics-specific content sessions focused on developing conceptual understanding of mathematics they teach as well as grade-level appropriate manipulatives.

The need to prepare students for a workforce with skills in science, technology, engineering, and mathematics (STEM) is growing and, in particular, computer science (CS; Computer Science Teachers Association [CSTA], 2016; National Research Council [NRC], 2012; National Science and Technology Council, 2018). The U.S. government's 5-year strategic plan for STEM education outlinds a commitment to equity and diversity, the need for transdisciplinary learning in which students develop mathematics literacy in meaningful and applied contexts, and the need to advance computational thinking as a critical skill (National Science and Technology Council, 2018).

Students are often first exposed to CS in high school; however, not all high schools include a CS course. Furthermore, females and minority students are underrepresented in these courses and in the workplace (CSTA, 2016; National Science and Technology Council, 2018). Earlier exposure to CS education at the K-8 level can help increase enrollment and lifetime engagement in CS for all students.

Embedding computational thinking (CT) practices within mathematics and science curriculum, instruction, and assessment provides opportunities to better prepare students as creative and critical thinkers to meet the future needs of the job market (Grover & Pea, 2013; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; NRC, 2012). For students to prepare for successful careers, they need to move beyond mathematics and science curriculum that focuses purely on the facts of each field. Teachers need to be prepared to address a multidisciplinary approach that incorporates mathematics, computing, and sciences for success in today's STEM fields.

Mathematical content and practices from the Common Core State Standards (CCSS) for Mathematics can be aligned with CS and CT practices at the middle school level. Integrating algorithmic and CT can be a meaningful way to emphasize the four C's needed to meet the 21st-century challenges: critical thinking and problem solving; communication; collaboration; and creativity and innovation (CSTA, 2016; International Society for Technology in Education [ISTE], 2018).

A specific challenge at the middle school level (grades 6-8) is that classes are often subject specific, and well-designed CS integration within core classes can be problematic (CSTA, 2016, p. 32). Additionally, CS concepts and CT skills that are outlined in current standards are not only new to students but also teachers, administrators, and parents (CSTA, 2016; ISTE 2018). Basic computer literacy activities such as creating documents or presentations and searching the internet are often incorrectly labeled as computer science (CSTA, 2016).

Within this context, we developed a summer institute aimed at addressing the need for high quality professional development in CCSS-Mathematics for middle school mathematics teachers with the goal of improving their content and pedagogical strategies in the context of CS. Called Coding for the Core: Computational Thinking and Middle Grades Mathematics, the institute incorporated a programming package developed by Code.org and Bootstrap Algebra that requires students to write code using algebra and geometry (Bootstrap:Algebra, n.d.; Schanzer et al., 2015). Additionally, programming robots in the Lego® Mindstorms® environment allowed for writing code emphasizing ratios and proportions as well as data analysis, statistics, and probability, among other topics (Carnegie Mellon University, 2019; LEGO Education, 2019).

This paper describes our investigation of teachers' experiences as participants in the institute in which we examined the following questions:

How does participation in comprehensive professional development including computer programming with Bootstrap Algebra and Lego Mindstorms robotics, mathematics content sessions, and mathematics pedagogy sessions impact:

- teacher understanding of middle grades mathematics content as it applies to computational thinking?
- teacher understanding of mathematics-specific technological pedagogical content knowledge (also known as technology, pedagogy, and content knowledge, or TPACK)?

We describe the design of the institute and our analysis of teacher performance on a mathematics and CT exam, changes in TPACK, and participant reflections about how the institute impacted their knowledge of mathematics and ways to integrate CT with an emphasis on coding or programming into instruction.

Literature Review

National Standards Emphasize Change

Both the *Next Generation Science Standards* (NGSS; NGSS Lead States, 2013) and the CCSS-Mathematics emphasized computational thinking practices (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; National Research Council, 2012). The NGSS included using mathematics and computational thinking as one of the eight recommended science and engineering practices. Furthermore, the CCSS-Mathematics emphasized eight Standards for Mathematical Practice, including making sense of problems and perseverance, reasoning abstractly and quantitatively, constructing viable arguments and critiquing the reasoning of others, modeling with mathematics, using appropriate tools strategically, attending to precision, looking for and making use of structure, and expressing regularity in repeated reasoning.

The CCSS-Mathematics were developed to address the need to prepare students for college and career expectations based on observations that, nationally, academic progress has been stagnant and there is a high need for mathematics remediation at the college level (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Key shifts from previous standards included a greater focus on fewer topics, coherent progressions from grade to grade, rigor with a pursuit toward conceptual understanding, procedural skills and fluency, and application of mathematics to real-life scenarios.

The ISTE standards for educators and students emphasized computational thinking competencies (ISTE, 2016, 2017, 2018).

Similarly to how technology is used by educators to deepen content area learning while building digital learning skills, teachers can integrate CT practices in their instruction to introduce computational ideas. This will enhance student content knowledge and build confidence and competence in CT. (ISTE, 2018, p. 1)

The CSTA (2016) developed the K-12 CS Framework based upon five core concepts with benchmarks provided for K-2, 3-5, 6-8, 9-10, and 11-12, together with five crosscutting concepts or themes integrated with the core concept statements and seven core practices that demonstrate engagement with the core concepts. As written, this framework serves as a conceptual guide for developing standards for independent CS courses or for integration with mathematics, science, and other subjects throughout the K-12 path.

Of the seven core practices, four are specific CT practices, including recognizing and defining computational problems, developing and using abstraction, creating computational artifacts, and testing and refining computational artifacts. Three of the practices are general practices of CS that support CT, including fostering an inclusive computing culture, collaborating around computing, and communicating about computing.

Disciplinary Computational Thinking

CT is defined by some as the "thought processes involved in formulating problems and their solutions so that the solutions are represented in a form that can be effectively carried out by an information processing agent" (Wing, 2011, p. 20). CT is a human ability characterized by problem solving, designing solutions, communicating thoughts in a creative, organized way, and debugging thoughts with or without a computer (CSTA, 2016, p. 69). Consensus has been achieved for the need to nurture and develop CT natives. In addition CT has gained enough importance for a suggestion to make "'rithms," short for algorithms, the fourth "r" for 21st-century literacy (Barr & Stephenson, 2011; CSTA, 2016; Grover & Pea, 2013; ISTE, 2016).

Current research endeavors have recognized the need to contextualize CT within specific disciplines (Gadanidis et al., 2017; Grover & Pea, 2013; Qin, 2009; Weintrop, et al., 2016; Yaşar, 2013). Benefits of embedding CT into mathematics and science classrooms include addressing the changing nature of these disciplines, such as bioinformatics and computational statistics, in the professional world; developing the reciprocal relationship between computational contexts and science and mathematics learning; and addressing the issues of underrepresentation of women and minorities in computer science fields (Weintrop et al., 2016).

Weintrop et al. (2016) developed a computational thinking in mathematics and science practices taxonomy that includes four major categories: data practices, modeling and simulation practices, computational problem-solving practices, and systems thinking practices. Each of these categories is composed of a subset of five to seven practices that are interrelated and dependent upon one another. Data practices include collecting, creating, manipulating, analyzing, and visualizing data. Modeling and simulation practices include using models to understand a concept, using models to find and test solutions, assessing models, designing models, and constructing models. Computational problem-solving practices include preparing problems for computational solutions, programming, choosing effective computational tools, assessing different approaches/solutions to a problem, developing modular computational solutions, creating computational abstractions, and troubleshooting and debugging. Systems thinking practices include investigating a complex system as a whole, understanding the relationships within a system, thinking in levels, communicating information about a system, and defining systems and managing complexity.

Theoretical Framework

The TPACK Framework serves as the guiding theoretical framework for this project. Participants were challenged to use this framework as a guide as they intentionally incorporated computational thinking practices into their curriculum, instruction, and assessment. As described by Mishra and Koehler (2006), the TPACK framework explores how technology is integrated with teaching through the following seven categories: technology knowledge (TK), content knowledge (CK), pedagogy knowledge (PK), pedagogical content knowledge (PCK), technological pedagogical knowledge (TPK), technological content knowledge (TCK), and TPACK.

The TPACK framework builds on the work of Shulman (1986) and is based upon the need for teachers to build subject-specific PCK. Koehler et al. (2013) emphasized the context-specific nature of incorporating digital technology by stating, "Integration efforts should be creatively designed and structured for particular subject matter ideas in specific classroom contexts" (p. 14).

Studies Incorporating TPACK Mathematics

Technology-based professional development for teachers should provide explicit opportunities for teachers to connect technology to curriculum, assessment, and instruction rather than teach technology skills in isolation (Hughes, 2005; Ndongfack, 2015). To prepare their students for the technological demands of the 21st-century, teachers need to develop the same skills as the students, which calls for sustained professional development opportunities (Goode et al., 2014; Matherson et al., 2014). To develop TPACK, teachers need opportunities to collaborate, plan, and reflect on their experiences on both learning and teaching with technology (Goode et al., 2014; Olofson et al., 2016).

Niess et al. (2009) described a five-stage Mathematics Teacher TPACK Developmental Model through which teachers can progress when they are learning to integrate technology in teaching and learning mathematics. This process begins when a teacher has sufficient PCK. As the teacher progresses through the stages using a particular technological tool TPACK is developed. These levels are as follows:

1. Recognizing (knowledge), where teachers are able to use the technology and recognize the alignment of the technology with mathematics content yet do not integrate the technology in teaching and learning of mathematics.
2. Accepting (persuasion), where teachers form a favorable or unfavorable attitude toward teaching and learning mathematics with an appropriate technology.
3. Adapting (decision), where teachers engage in activities that lead to a choice to adopt or reject teaching and learning mathematics with an appropriate technology.
4. Exploring (implementation), where teachers actively integrate teaching and learning of mathematics with an appropriate technology.
5. Advancing (confirmation), where teachers evaluate the results of the decision to integrate teaching and learning mathematics with an appropriate technology. (p. 9).

Descriptors and examples are provided for the Mathematics Teacher TPACK Development model for four themes, including curriculum and assessment, learning, teaching, and access.

Richardson (2009) described a professional development project for 20 eighth-grade Algebra I teachers focused on developing their TPACK knowledge by integrating technology such as TI-Nspire, virtual manipulatives, and GeoGebra. The teachers' exit interviews indicated that they advanced in their development of TPACK; however, they needed more training in making a transition from using technology-based manipulatives for illustrating content to using these tools to help students develop conceptual understanding of concepts. Common challenges included helping teachers focus on technology, content, and pedagogy collectively rather than focusing solely on the technology in and of itself.

Olofson et al. (2016) defined TPACKing, which is "an active process carried out by the teacher in which s/he constructs knowledge in the technology-rich setting" (p. 189). This process is continuously modified because teachers draw upon context, experience, and knowledge of students as they build TPACK. Then, as they enact TPACK in their unique setting with students, they change their TPACK construct. This change influences their understanding and beliefs about technology, pedagogy, and content.

Olofson et al. (2016) used a multiple case study method and a radical constructivist lens to isolate four unique ways in which teachers develop TPACK, including interpersonal, environmental, and internal interactions and equilibration. The authors suggested that the TPACKing framework could be used during professional development to help analyze in-service teachers' progress, to identify practices that lead to TPACKing and how making the process explicit to teachers impacts their constructions.

Professional Development Design

Coding for the Core provided professional development for 22 middle school teachers (17 mathematics, two science, and three STEM), ranging from 1 to 36 years of experience, focused on computer programming and robotics, with specific connections to CCSS-Mathematics. The project was funded through the state of Tennessee Improving Teacher Quality (ITQ) grant program and included a 2-week summer institute held at a rural university in Tennessee along with two follow-up Saturdays during the fall semester for a total of 72 contact hours.

This institute aimed to improve teachers' mathematics content and pedagogical knowledge, with a focus on best practices for instruction as required by the Tennessee Educator Acceleration Model, or TEAM, which is the evaluation system required by the state (Tennessee Department of Education, 2019). Teacher participants received a $75 daily stipend, a Lego Mindstorms EV3 Core Set, hands-on mathematics manipulatives, and publications geared for middle level teachers on the topics of robotics and hands-on manipulatives.

Coding for the Core used a programming package developed by Code.org and Bootstrap (Schanzer et al., 2013) that requires students to write code that illustrates algebra and geometry concepts. Additionally, programming robots created using Lego Mindstorms allowed for writing code illustrating ratios and proportions as well as data analysis, statistics, and probability. The goal was to promote a change in teacher understanding of mathematics and TPACK with the use of CT practices emphasizing computer programming activities and additional instruction using hands-on and virtual manipulatives directly aligned with middle school content.

Professional Development Framework

The summer institute was planned using characteristics of effective professional development, which included the following components: focused on clear goals, based on content and practice, provided active learning experiences, led by facilitators with appropriate expertise, aligned with state and district goals and standards, and enabled collaborative and collective participation of teams of teachers (Demonte, 2013; Desimone, 2009; Koba et al., 2013; Loucks-Horsley et al., 1996). Although we provided 72 contact hours over 4 months, due to the nature of the ITQ grant, we were unable to extend the grant for a longer period of time, which is a limitation of this study.

Coding for the Core was conducted jointly by education methods and mathematics and computer science faculty to effectively model pedagogy and focus on building mathematics content knowledge and practices within the context of embedding CT practices. This approach aligned with a principle of effective PD for mathematics and science education that suggests the importance of providing "teachers with opportunities to develop knowledge and skills and broaden their teaching approaches, so they can create better learning opportunities for students" (Loucks-Horsley et al., 1996, p. 1).

CT practices were directly linked to CCSS-Math standards for mathematics content and mathematics practices. Participants were engaged in TPACKing (Olofson et al., 2016) as they reflected upon their individual contexts, experiences, and knowledge of students and developed their beliefs about technology, pedagogy, and content. Participants were also engaged in active learning throughout each day using computers, robotics, and mathematics manipulatives.

As recommended by Loucks-Horsley et al. (1996), teachers worked in learning communities and were prepared to serve in leadership roles. During the workshop teachers worked in groups of three to four in grade-level teams as they experienced activities that embedded CT practices that mirrored methods they could use with students. Project staff explicitly addressed teacher's knowledge of TPACK within mathematics as well as the context of CT practices.

Each participating middle school identified at least two mathematics teachers to form a professional learning community (PLC). In some cases when only one mathematics teacher volunteered, we recruited a science or STEM teacher. These PLCs were also challenged to seek opportunities within their school and district to provide at least one professional development session for their peers.

Institute Schedule and Design

Our schedule typically allowed for four blocks of instruction each day, including 90 minutes each for Bootstrap Algebra Units and Lego Mindstorms challenges led by the mathematician/computer scientist on our team, 2 hours for mathematics-specific content aligned with Bootstrap and Lego units led by our mathematician, and 1 hour for pedagogical strategies emphasized by TEAM (Tennessee Department of Education, 2019) led by our mathematics education specialist. Day 1 allowed for teachers to complete their preassessment content test, the TPACK survey, and a baseline TEAM survey. The teachers also completed an Hour of Code at code.org and an icebreaker in which they formed teams to construct artistic robots made using a plastic cup, a battery, wires, a motor, and markers. They were introduced to Lego Mindstorms, received their Lego kits, and had an opportunity to sort their bricks. Following Day 1 the teachers followed four blocks of instruction.

Even though the taxonomy developed by Weintrop et al. (2016) was published after the summer institute, its framework of practices (data, modeling and simulation, computational problem-solving, and systems thinking) was useful in describing our institute design. In the data practices category, Lego Mindstorms robots were used to generate experimental data to illustrate concepts including ratio and proportion, rates of change, and statistical concepts. These data were then manipulated and displayed graphically and the results were analyzed.

The Bootstrap programming also involved data practices. To test the code the participants were required to generate test data. In the modeling and simulation practices category, functional models were created using the data collected from the Lego Mindstorms robots, and these models were tested. The Bootstrap programming also involved developing functional models of the motion of game components.

In the computational problem-solving category, both the Lego Mindstorms and the Bootstrap activities involved computer programming. This programming often involved breaking the problem down into functional modules and then creating computational abstractions to create these modules.

Finally, the programs needed to be tested and debugged. The final category in Weintrop's taxonomy is systems thinking practices, which were addressed in both the Lego Mindstorms and the Bootstrap programming. A robot, together with its programming, is a complex system, as is a computer game developed in Bootstrap. The relationships between the various components must be understood and managed for the system to operate correctly.

Block 1: Bootstrap Algebra

Bootstrap Algebra is a 20- to 25-hour module divided into nine units specifically focused on using computer science in algebra to construct a videogame around three elements: a player (the user's avatar), a target (something the player wants), and a danger (something the player must avoid). Each unit in the Bootstrap curriculum is designed to integrate and introduce three interrelated components: a new game feature, a programming concept, and a mathematics concept (Schanzer et al., 2015).

For example, in Unit 1 the game feature is locating elements on a screen, the programming concept is creating expressions with the use of "Circles of Evaluation," and the mathematics concept is cartesian coordinates. The video game is built in a sequence of frames, and a function is written to locate each character within a frame to describe the character's change in position as the character moves. The Bootstrap curriculum allows for students to model three representations of functions: symbolic form, domain and range, and input/output tables (Schanzer et al., 2018).

Bootstrap Algebra begins with learning how to diagram expressions and practice writing functions using a notation called Circles of Evaluation. This notation provides an organized means to express the order of operations. We asked the teachers to transform the circles into a "syntactically valid textual code" called Scheme using the Racket programming language (Schanzer et al., 2015, p. 2). The circles of evaluation provide a means to clearly connect algebraic functions and expressions to formal notation or written computer code.

Bootstrap Algebra also used a "Design Recipe" for solving word problems and designing functions. The design-recipe worksheet requires programmers to define the domain and range and write examples of the results of their function in action. This strategy provides an organized means to write and test code (Schanzer et al., 2015). As previously noted, this block addressed the four categories in Weintrop's et al. (2016) taxonomy of CT practices. The participants were creating a complex system (systems thinking practices) by breaking the problem into functional modules that were then created using computer programming (computational problem-solving). Many of these modules modeled the motion of objects on the computer screen (modeling and simulation) and needed to be tested using data created by the participants (data practices).

Block 2: Lego Mindstorms

Prior to introducing challenges with the Lego Mindstorms EV3 Education Core set, we spent several days introducing the software and sensors using Carnegie Mellon Robotics Academy's Introduction to Programming curriculum modules (Carnegie Mellon University, 2019). We followed these introductory lessons with robotics teaching modules modified and presented as challenges from the Rensselaer Polytechnic Institute's Center for Initiatives in Pre-College Education (http://www.rpi.edu/dept/cipce/index.html).

These lessons were designed for use with the Lego Mindstorms NXT brick and software; however, we modified them as needed to use with the EV3 brick and software. The lessons included Reaction Time, How Fast is that Robot?, What is my Rate?, The Wheels on the Robot: Pi Day, Mars Rover: Follow Your Curiosity, The Chance Dance, and Random Robot Racers. Table 1 includes a description of the Lego Mindstorms EV3 components used as well as the mathematics components emphasized within each lesson.

This block also addressed Weintrop's et al. (2016) four categories of computational thinking. In many of the activities the participants were creating a complex robotic system consisting of both hardware and software to address a problem (systems thinking). Their task involved programming and assessing different approaches to the problem (computational problem-solving). In many of the activities this robotic system was then used as model to generated data illustrate a mathematical concept (data practices). A functional model was then created from this data (modeling and simulation).

Block 3: Mathematics Content Session

Our mathematician led teacher participants through 2 hours daily of mathematics-specific content aligned with the Bootstrap and Lego units as well as other content that is specifically challenging at the middle school level. These activities incorporated the use of hands-on manipulatives and the eight CCSS-Mathematics practices, both of which the majority of the participants stated they did not use or emphasize on a regular basis.

Table 1
Lego Mindstorms EV3 Components and Mathematics Concepts by Lesson

Lego Mindstorms EV3 Lesson	Lego Mindstorms EV3 Components	Embedded Mathematics Concepts
Reaction Time	Touch Sensor	Mean, Median, and Mode. Record reaction times for no distraction, listening to music, and texting.
How Fast Is That Robot?	Driving Base	Compare ratios of times to ratios of distances; setup proportions to predict distances; calculate speed and use to predict distances
What Is My Rate?	Driving Base	Calculate distance and velocity; discuss linear relationships, interpolation, and relative error.
The Wheels on the Robot: Pi Day	Driving Base Different sized wheels	Graph relationships of wheel size and distance traveled; calculate the ratio of the wheel size and distance traveled over one rotation; discuss pi and its significance.
The Chance Dance	Driving Base	Theoretical and empirical probability
Random Robot Racers	Driving Base	Compute the mean absolute deviation (MAD) for each program; compute theoretical MAD for each program and compare with empirical results.

The sessions were interactive with opportunities for discussion and questions focused on developing conceptual understanding of the content. We put participants in the position of "teacher as student/learner," in which they were held accountable for the eight CCSS-Standards for Mathematical Practice through making sense of problems and perseverance, reasoning abstractly and quantitatively, constructing viable arguments and critiquing the reasoning of others, modeling with mathematics, using appropriate tools strategically, attending to precision, looking for and making use of structure, and expressing regularity in repeated reasoning.

The teachers were led through a series of problem-based tasks each day with a conscious effort to meaningfully integrate manipulatives. Lesson 1 introduced solving problems using combination charts. The teachers solved several problems adapted from *Rich and Engaging Mathematical Tasks: Grades 5-9* (Lappan et al., 2012, pp. 167-175), which led to writing and solving a system of linear equations. The system was solved through graphing and the use of *X-Y* coordinate pegboards, which were new instructional tools for all of the workshop participants. These pegboards enable graphing through the use of colored pegs representing points in the coordinate plane. The pegs can be connected using rubber bands to graph lines and other geometric shapes.

Lesson 2 focused on working with rational numbers through several problems adapted from *Bits and Pieces II* (Lappan, et al., 2006, pp. 43-45). This lesson focused on adding and subtracting fractions of land using a grid/area model. Lesson 3 focused on multiplying fractions using an area model as a means to explain the standard multiplication algorithm. Lesson 4 focused on linear equations with the use of dot patterns, creating a table and graph or scatterplot on paper as well as on *X-Y* coordinate pegboards to explore sequences both graphically and as formulas.

Lesson 5 focused on measures of central tendency, particularly the mean, through a problem focused on the length of the first names of all workshop participants adapted from *Data About Us* (Lappan, 2002). Participants were led to use cubes that could be snapped together (Unifix or snap cubes) to make a tower whose height was the same as the number of letters in their first name and then trade cubes with members in a small group until all of the towers were the same height.

Lesson 6 again focused on systems of linear equations. This time linear expressions and equations were modeled using ETA hand2mind Algeblocks®, a three-dimensional set of geometric prisms that model two-variable polynomial expressions (ETA hand2mind, 2019). While some teachers said that they had access to Algeblocks in their schools, they said they did not know what they were or how to use them prior to participation in the workshop.

Lesson 7 explored ratios and proportions with Cuisenaire rods through activities adapted from *Hands-on Standards, Common Core Math* (ETA hand2mind, 2012) and *Teaching the Common Core Math Standards with Hands-on Activities* (Muschla, et al., 2012). Cuisenaire rods are a set of 10 rods, each a different color that increase in length from 1 to 10 centimeters. The participants were asked to use multiple combinations and trains of rods to model ratios and proportions.

Lesson 8 explored two different uses of Barbie® dolls for teaching middle school mathematics. First, participants were asked to measure the length of Barbie's arms, legs, and torso and compare these measurements to the same body parts of volunteers in the group to determine if Barbie's body proportions were realistically portrayed. Second, participants worked in teams to determine how long to make a bungee cord so that the first time they dropped Barbie, her hair touched the ground but the head did not hit. This lesson challenged them to graph data to establish a linear pattern, develop an equation to model the simulation, use an equation to predict the appropriate length of bungee cord required, and test their predictions (AIMS Education Foundation, 2006).

Block 4: Mathematics Pedagogy Session

Our mathematics education specialist concluded each day of the summer institute with 1 hour of discussion and activities focused on developing TPACK and best practices for instruction as emphasized by the Tennessee Educator Acceleration Model, or TEAM, which is the evaluation system required by the state (Tennessee Department of Education, 2019). During the first session teachers worked in grade-level teams to brainstorm ways in which they addressed the 12 instruction indicators of the TEAM rubric. They added specific details to posters around the room, which were titled by each TEAM indicator for instruction: standards and objectives, motivating students, presenting instructional content, lesson structure and pacing, activities and materials, questioning, academic feedback, grouping students, teacher content knowledge, teacher knowledge of students, thinking, and problem solving.

The grade-level teams contributed specific strengths and challenges that they perceived with addressing each indicator and added a summary of their ideas to each poster. They were asked to explicitly reflect upon the Bootstrap Algebra modules, Lego Mindstorms EV3 modules, and mathematics content sessions each day and add what they were learning to the posters. This was an attempt to engage in TPACKing by making connections between technology, content, and pedagogy (Olofson et al., 2016).

Additional activities discussed and modeled with the mathematics education specialist were integrating CCSS-Standards for Mathematical Practices and tasks, classroom discussion strategies/mathematics talks, formative assessment classroom techniques, differentiated instruction, classroom management strategies, and instructional technology integration (e.g., Plickers, Kahoot, and Screencasting). Each teacher received the following texts from which we modeled at least one activity and or discussed the content:

- *The Lego Mindstorms EV3 Discovery Book: A Beginner's Guide to Building and Programming Robots* (Valk, 2014),
- *Classroom Activities for the Busy Teacher: EV3* (Kee, 2013), *Differentiating Instruction With Menus for the Inclusive Classroom: Math Grades 6-8* (Westphal, 2013),
- *Teach Like a Champion 2.0: 62 Techniques That Put Students on the Path to College* (Lemov, 2015),
- *Mathematics Formative Assessment: 75 Practical Strategies for Linking Assessment, Instruction, and Learning* (Keeley & Tobey, 2011), and

- *Hands-on Standards, Common Core Math* (ETA hand2mind, 2012).

The teachers were also given time to discuss and make a wish list of their favorite mathematics manipulatives used in the mathematics content sessions and highlighted in the mathematics texts they received.

Saturday Workshops: Fall Semester

The two Saturday workshops in the fall semester were used to distribute hands-on manipulatives that the teachers requested from their wish lists, plan and troubleshoot lesson plans incorporating robotics and mathematics-specific manipulatives in their classrooms, plan a professional development session to share what they learned at their respective schools or in their school systems, and take postassessments. Their top choices for mathematics manipulatives included *X-Y* coordinate pegboards, Algeblocks (ETA hand2mind, 2019), and Cuisenaire rods.

Several teachers were able to share strategies and tools used in the training with teachers at their own schools as part of formal professional development sessions and many more in informal discussions with colleagues. Two sixth-grade teachers, one mathematics and one science at the same school, began a robotics club after school; several teachers were able to integrate the use of the Lego Mindstorms robotics as part of robotics teams that were already established in their communities; one teacher used Lego robotics as part of daily enrichment classes with his seventh-grade students; one grades 6-8 STEM teacher used the Bootstrap Algebra programming with his eighth-grade students and used Lego Mindstorms robots on a regular basis in all STEM classes; one grades 6-8 STEM teacher, new to the field, began using Lego Mindstorms robots with students in her classes; and two teachers (one sixth-grade and one eighth-grade at same school) began using *Hour of Code, Beyond the Hour* from Code.org to engage students during their weekly computer lab mathematics classes.

Method

This project was designed using a mixed methodology approach of collecting qualitative and quantitative data because both types of data had equal value for understanding the research questions (Buchholtz, 2019; Creswell & Clark, 2017). A convergent parallel design was used to collect both types of data concurrently (Creswell & Clark, 2017). Quantitative data were collected using a mathematics content and CT assessment designed specifically for this institute by grant staff as well as the Survey of Preservice Teachers' Knowledge of Teaching and Technology (Schmidt et al., 2009).

The specific items administered from this TPACK survey included Likert-scale items addressing TK (Questions 1-6; n = 6), CK Mathematics (Questions 7-9; n = 3), PK (Questions 19-25; n = 7), PCK Mathematics (Question 26; n = 1), TCK Mathematics (Question 30; n = 1), TPK (Questions 34, 35, 37-39, 41, 42; n = 7) and TPACK Mathematics (Question 43; n = 1). Questions 36 and 40 were not used, as they dealt specifically with teacher education programs. In addition, Questions 47-57 were not used because they dealt with the teacher education program and cooperating teachers. Quantitative data was analyzed using two-sample *t*-tests with the use of a Bonferroni correction to determine the statistical significance of changes.

Narrative analysis was used to discover emergent themes within the qualitative data collected pre- and postparticipation (as recommended in Patton, 1990). Participant responses to one of the three open-ended prompts included on the TPACK survey, as well as responses to a final project evaluation form designed by project staff served as the qualitative data.

The TPACK survey prompt was provided pre- and postparticipation and asked participants to describe a specific teaching episode in which they effectively demonstrated or modeled combining content, technologies, and teaching approaches in a classroom lesson. The final project evaluation form required participants to describe the "top three take-aways" from participation, the most helpful part of training, and what could have been done to improve their experiences. The responses to the TPACK survey open-ended prompt were categorized into teacher-focused and student-focused use of technology, and themes were identified for the types of technology integration that the teachers described.

The responses to the project evaluation form were analyzed to search for similarities and differences between participant ideas in order to identify the emergent themes for top take-aways, what was most helpful and what could have been improved. These themes were also compared with teacher responses to the TPACK survey and observations were made about how teachers changed their views and actions in the classroom as a result of grant participation.

Findings

This section reports a comparison of teacher performance on the mathematics and computational thinking pre- and postassessment, both the quantitative and qualitative results for the TPACK survey, and both quantitative and qualitative analysis of the Coding for the Core final evaluation form.

Mathematics Content and Computational Thinking

The content assessment designed by grant staff consisted of 15 questions that integrated CT skills and mathematics common core content with grades 6-8 mathematics. All 22 teachers took both the pre- and postassessment. The pretest average was 60, with a range in scores from 0 to 86.7. The posttest average was 73.9, with a range of scores from 23-100. The 13.94 increase in the average scores was statistically significant at the $p < 0.0001$ level with the use of a two-sample *t*-test.

The assessment included a mixture of question types that represent a range of the practices, as developed by Weintrop et al. (2016). To determine which questions had the largest contributions to the overall statistical significance, individual pre- and postassessment two-sample *t*-tests were performed. When a Bonferroni correction was applied to this collection of 15 individual *t*-tests, only two of the questions, 11 and 15, were independently statistically significant.

To illustrate their contributions, five out of the 15 questions were selected for discussion here. These questions include 3, 4, 7, 11, and 15. Figures 1-5 illustrate each question, the average score on the pre- and postassessments, the significance level of the increase, the mathematics content assessed, and the computational thinking category assessed. Again, note that the significance levels reported here may not show that the questions are independently statistically significant, but they do illustrate the relative contributions of each question to the overall significant result.

Question 3, illustrated in Figure 1, measured each participant's ability to select a real-world context and design a method to collect data that could be analyzed using measures of central tendency, mean, and median, and to analyze the data to make a statement about the results. This question aligned directly with the Lego Mindstorms lesson called Reaction Time, in which participants collected and analyzed data using the touch sensor and calculated mean, median, and mode for no distraction, listening to music, and texting.

They also used Unifix cubes within a mathematics content session to develop conceptual understanding of mean, median, and mode. This particular assessment question in conjunction with Question 2 required them to use the computational thinking practice category of data practices, including collecting, creating, manipulating, analyzing, and visualizing data.

Question 2 was given for reference. Question 3 addressed the component of the standard concerned with generating "multiple samples of the same size to gauge the variation in estimates or predictions." The average score for Question 2 on the pretest was 80%, and the average score on the posttest was 86%. The average score for Question 3 was 45% on the pretest and 64% on the posttest. The difference in performance between Question 2 and 3 points to the relative ease with which the teachers calculated mean and median and described a conclusion for Question 2, compared to the more challenging task in Question 3 of designing their own method to collect similar data.

Questions 4 and 7, illustrated in Figures 2 and 3, respectively, focus on writing, interpreting, and evaluating numerical expressions as well as the order of operations. These questions incorporated the circle of evaluation format for diagramming expressions and practice writing functions used in the Bootstrap Algebra curriculum. As noted, the circle of evaluation helps provide a means to connect algebraic functions and expressions to formal notation or written code.

Figure 1 Content and CT Assessment Problem 3

3. Describe a method that could be used to generate real world data similar to that in the table in problem 2 in class. (7.SP)

Problem 2 (for reference)

2. Consider the reaction times for students who are distracted by music vs a control group of students who do not have this distraction. Calculate the mean and median for both groups and draw a conclusion. (7.SP.2,4)

	Reaction Times (sec)					
Control	0.3	0.2	0.4	0.2	0.4	0.5
Music	0.3	0.4	0.6	0.3	0.3	0.7

Mean (Control) = ________

Median (Control) = ________

Mean (Music) = ________

Median (Music) = ________

Conclusion:

Mathematics Content

7.SP.A.2 - Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Pre-Test 45%

Post-Test 64%

p-value: .014

Computational Thinking Category

Data Practices

These particular assessment questions required participants to use the computational thinking practice category of computational problem solving, including creating computational abstractions and developing modular computational solutions. Despite never using the circle of evaluation prior to grant participation, a large percentage of the teachers were able to answer Questions 4 and 7 correctly on the pretest. After receiving instruction with the Bootstrap Algebra curriculum, they showed an improvement for each question.

Questions 11 and 15, illustrated in Figures 4 and 5, respectively, focus on the mathematical function definition. In particular, several of the activities in the Lego Mindstorms, Bootstrap, and mathematics content sessions were aimed at thinking about functions in a more abstract manner not limited to numerical computations. The understanding that functional relationships, the ability to map input values to results, are not limited to numerical calculations is critical in CT. This application of functions is significant in shifting from mathematical thinking to computational thinking.

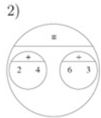
Question 11 was particularly relevant to the Bootstrap programming and required the participants to identify the domain and range of a function when the result was a graphical element rather than a numerical value. This task required them to use the CT practice category of modeling and simulation, including using models to understand a concept and assessing models. The participants were clearly not familiar with this type of nonnumerical function before the workshop. They displayed a dramatic improvement on this problem after the workshop.

Figure 2 Content and CT Assessment Problem 4

4. One way to indicate operations in an expression is to draw a circle of evaluation. For example $4 - 5$ could be drawn

Using this convention do the following:

(a) Convert the following circle of evaluation into an arithmetic expression. (5.OA.1-2)



(b) Draw a circle of evaluation for the expression $3 * (4 - 6)$. (5.OA.1-2)

Mathematics Content

5.OA.A.1-2 - Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them

Pretest 84%

Posttest 98%

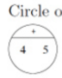
p-value: .041

Computational Thinking Category

Computational Problem Solving

Figure 3 Content and CT Assessment Problem 7

7. Suppose we represent mathematical operations in three different forms

Traditional Form	Circle of Evaluation	New Form
$4 + 5$		$(+\ 4\ 5)$

Notice that this new form emphasizes that the operation $+$ is really a function that accepts the arguments 4 and 5 and then performs the "addition" operation to produce the output 9.

Using this notation evaluate the following: (5.OA.1-2)

(a) $(-\ 4\ 6)$

(b) $(-\ 5\ (+\ 2\ 3))$

Mathematics Content

5.OA.A.1-2 - Use parentheses, brackets, or braces in numerical expressions and evaluate expressions with these symbols.

Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them.

Pretest 80%

Posttest 93%

p-value: .0053

Computational Thinking Category

Computational Problem Solving

Figure 4 Content and CT Assessment Problem 11

11. Suppose a function takes a color (“red”, “blue”, or “green”) and a size (positive integer) and draws the image of a square of the given size and color on the computer screen. Identify the domain and range of this function. (F-IF.1) Mathematics Content F-IF.A.1 - Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	Pretest 16% Posttest 61% *p*-value: .00013 Computational Thinking Category Modeling & Simulation

Question 15 also required the participants to identify the domain and range of two functions. In Part A the function was numerical, while in Part B the function again involved a graphical element. In addition, this problem required more abstraction than did Question 11 because the answer needed to be specified as a “function contract,” mapping the domain to the range.

Creating this type of function is key to developing modular computational solutions and creating computational abstractions. It also illustrates that functions are not limited to mappings between sets of numbers but can include mappings between ordered tuples of numbers, graphical images, colors, or strings. Thus, this question addresses the CT practice category of computational problem solving. As with Question 11, the participants were not comfortable with nonnumerical functions before the workshop and displayed a marked improvement after the workshop.

Survey of Preservice Teachers’ Knowledge of Teaching and Technology

TPACK Survey Quantitative Results

A subset of 26 Likert-scale items form the Survey of Preservice Teachers’ Knowledge of Teaching and Technology (Schmidt et al., 2009) were administered to the participants. Of the 22 participants, 17 completed both the pre and post TPACK survey. The specific items administered from the TPACK survey included Likert-scale items addressing TK (Questions 1-6; n = 6), CK Mathematics (Questions 7-9; n = 3), PK (Questions 19-25; n = 7), PCK Mathematics (Question 26; n = 1), TCK Mathematics (Question 30; n = 1), TPK (Questions 34, 35, 37-39, 41, 42; n = 7) and TPACK Mathematics (Question 43; n = 1).

Each item response was scored with a value of 1 assigned for *strongly disagree* to 5 for *strongly agree*. The participants’ responses were averaged over all 26 questions. Additionally, the participants’ responses were averaged over each construct. For example, the six TK questions were averaged to produce one TK score. A two-sample t-test was computed for the participants’ average responses over all the questions to show a significant change (p = .000746).

Figure 5 Content and CT Assessment Problem 15

<table>
<tr>
<td rowspan="2">
15. A computer deals with data that can take a variety of different forms. These include:
Number: Any real number (e.g. 4, 3.2, -7)
String: A list of characters inside double quotes (e.g. "red", "Susan", "Triangle")
Image: An image drawn on a computer screen. (e.g. a dog  or a triangle ▲)
When defining a function for the computer it is necessary to provide a function contract that states the function name, its domain and its range. If the domain has more than one element, they are separated by commas. For example
<code>Add: Number, Number -> Number</code>
is a contract for Add that takes two numbers and produces their sum.
Using this system, write a contract for each of the following functions. (F-IF.1)
(a) The Abs function that produces the absolute value of a number.
(b) The Circle function that draws the image of a circle when provided with a radius and the name of a color.

<u>Mathematics Content</u>
F-IF.A.1 - Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
</td>
<td>Pre-Test 27%

Post-Test 59%

p-value: .0025</td>
</tr>
<tr>
<td><u>Computational Thinking Category</u>

Computational Problem Solving</td>
</tr>
</table>

To determine the individual contributions, separate two-sample *t*-tests were performed on each construct. Once a Bonferroni correction was imposed, no individual construct showed a statistically significant change. However, the collective result was statistically significant. Table 2 includes the participant average results for the pre and post TPACK survey along with the *p*-value to help determine the contribution of each construct to the overall statistical significance.

TPACK Survey Qualitative Results

Of the three open-ended prompts on the TPACK survey, one was administered to the participants: "Describe a specific episode where you effectively demonstrated or modeled combining content, technologies and teaching approaches in a classroom lesson. Please include in your description what content you taught, what technology you used, and what teaching approach(es) you implemented."

Table 2
Pre and Post TPACK Survey Results

	Pretest		**Posttest**		
TPACK Subscale	**Mean**	***SD***	**Mean**	***SD***	***p* value**
TK (6 items)	3.56	0.56	3.74	0.56	0.049
CK Mathematics (3 items)	4.08	0.76	4.24	0.69	0.135
PK (7 items)	4.13	0.51	4.18	0.39	0.267
PCK Mathematics (1 item)	3.88	0.87	4.06	0.77	0.166
TCK Mathematics (1 item)	3.47	0.81	4.06	0.90	0.014
TPK (7 items)	3.79	0.56	4.13	0.27	0.008
TPACK Mathematics (1 item)	3.53	0.87	4	0.71	0.021

Of the 17 participants who responded to both the pre- and postsurvey, five described student activities in the presurvey that aligned with Weintrop et al.'s (2016) computational thinking in mathematics and science taxonomy, including two science teachers who had students use online simulations to explore open and closed circuits and Boyle's law (modeling and simulation practices); a mathematics teacher who asked students to use iPads to collect and analyze data (data practices), a mathematics teacher who had students program and troubleshoot robots in an afterschool program (computational problem-solving practices), and a mathematics teacher who asked students to create real-world structures for Minecraft to demonstrate their understanding of scale (systems thinking practices). Five teachers described teacher-focused use of technology for assessment purposes with the use of game-like practice in an electronic format in the presurvey. Six teachers described student-focused use of technology, including three mathematics teachers who described the students using graphing calculators to solve problems; one science teacher who had the students use a webquest to obtain proof for or against a hypothesis of "Dinosaurs had feathers"; one mathematics teacher who set up an interactive whiteboard for students to solve mathematics problems as part of station rotations; and one mathematics teacher who had the students use the internet to research ideas for balloon-powered cars for in-class

STEM projects. One mathematics teacher stated that she did not use technology for instruction in the presurvey.

In the postsurvey seven teachers described activities that aligned with Weintrop et al.'s (2016) computational thinking in mathematics and science taxonomy, with four of those specifically related to using robotics, two specific to coding/programming in the regular education classroom, and one use of coding in a related arts computer class. Table 3 includes a comparison of these seven teachers' pre- and postsurvey responses.

Additionally, five teachers described the use of Plickers for formative assessment and immediate feedback, as was emphasized in grant training (https://get.plickers.com). One of these teachers also emphasized the use of the "No Opt Out" strategy, in which teachers ensure that students who are not able to respond correctly to a prompt can answer correctly after another student has shared the answer from *Teach Like a Champion 2.0* (Lemov, 2015). Finally, two mathematics teachers described the advantage of students using manipulatives rather than the integration of technology as a result of grant participation.

Coding for the Core Final Project Evaluation Form

Nineteen of the 22 participants completed the final project evaluation form. Eleven Likert-scale questions were asked about their perceptions regarding preparation for teaching as a result of participation in the project. Three open-ended questions were asked to determine three take-aways gained from their experience, the most helpful aspects of the project, and what could have improved their experience in the project. Each Likert-scale item response was scored with a value of 1 assigned for *strongly disagree* to 5 for *strongly agree*. Table 4 includes the results of the 11 Likert-scale items.

Table 3
TPACK Survey Open-Ended Prompt Pre- and Post-Comparisons

Presurvey	Postsurvey
8th-grade mathematics, 1st year	
Teacher use of technology - game I used a Kahoot! to go over finding the interior angle sum of a regular polygon, this made the review of this topic game-like and helped students stay engaged.	*Teacher demonstration with robot (Modeling & Simulation; Computational Problem Solving)* I used the EV3 to model functions with the color sorter, and we talked about how the written code is in itself a function.

Presurvey	Postsurvey
Presurvey, 4th-grade mathematics; Postsurvey, 7th-grade mathematics & science, 6 years	
Student use of technology - calculator Students had to learn to use a calculator to add, subtract, multiply, and divide fractions. We had already learned how to do these concepts manually through modeling and solving. Since the students understood how to do this, we simply broke each step down and applied them to the calculator. I used my active board to pull up a large screen-shot of the same model of calculator and the white board to write the problems.	*Teacher demonstration with robot (Modeling & Simulation; Systems Thinking)* I used the Lego robot as a demonstration of how simple machines can even help a robot. We figured that the robot would not be able to climb over a book laying on the table. We added a ramp and the robot was easily able to drive up and over the book... We then expanded that into how many simple machines were used on the robot to make a complex machine; the robot itself. Since the robot was new to them, it grabbed their attention and kept them engaged. The same concepts could have been taught using any other comparisons like scissors, door stops, staplers etc. but the level of student engagement would not have been nearly as high.
Presurvey, 8th-grade honors physical science; Postsurvey, 6th-grade mathematics & science, 15 years	
Student use of simulation (Modeling & Simulation) When teaching Boyle's law I was able to effectively integrate a virtual lab into the lesson which allowed students to experiment with the variables before we began to look at determining how the variables were affected in formula form.	*Student use of coding (Computational Problem Solving)* I was able to use coding to enhance students' understanding of algebraic reasoning.
6th-grade mathematics, 1st year	
Student use of the internet Students created a car out of recycled materials that needed to be balloon powered. They were not given any instructions other than the could use one balloon, one straw, and recycled materials. Students were able to use the Internet to look up car ideas. Students participated in hands-on learning, used visual aids to decorate the car, and used math and science knowledge to determine how to use one balloon to make their car travel the farthest distance.	*Student use of programming (Computational Problem Solving)* I have used the code.org site in my classroom on a weekly basis for students. Students are using computer science and important math skills... I am able to help students strengthen their problem-solving strategies.

Presurvey	Postsurvey
8th-grade mathematics, 8 years	
Student use of technology - calculators I used graphing calculators to demonstrate how the different parts of a linear equation affect the outcome of the graph. Students were taught how to plug in equations into the calculator, graph them, manipulate the window sizes to fit the graph (focus on deriving an appropriate scale size based upon the components of the equations), and identify the impact of slope and y intercepts.	*Student use of programming (Computational Problem Solving)* I am co-teaching programming with our schools Encore teacher.
6th-grade mathematics, 9 years	
Teacher use of technology - Assessment & Screencast I often use a response system in my lesson to gauge whether or not students are following and understanding a lesson. I often make a movie or PowerPoint lesson with recorded text and explanations and take my students to a computer lab where they watch and follow the lesson and complete guided practice. The students really like it because they can back it up and replay parts they missed or did not understand. This is great for simple task-based concepts like mean, median, mode and range.	*Student use of robotics/programming (Data practices; Computational Problem Solving)* I was able to use the concepts from the lab we did to have my students determine the rate at which the probot cars travel. They are also working during enrichment times on problem solving by having the probot cars navigate a maze and other challenging tasks.
7th-grade mathematics, 3 years	
Student use of technology for design (Systems Thinking) With my students having access to one to one technology in the classroom, I find myself searching for new ways to incorporate that technology to enhance their learning. A specific example would be when we used Minecraft to recreate real world structures inside of Minecraft. This was a great way for the students to demonstrate their ability & knowledge dealing with scale.	*Student use of robotics/programming (Modeling and Simulation; Computational Problem Solving)* In order to secure funds for additional EV3 robots, our class put together a short video showing how we are using them in the classroom to enhance students' understanding of mathematical concepts. This included the use of the software to write certain codes to get our robots to travel through a maze.

Table 4
Final Project Evaluation Form Likert-Scale Items

Question Prompt	Average
I feel that I am better prepared to teach using TEAM pedagogy.	4.11
I feel that I am better prepared to use critical thinking and problem-solving activities in my classroom.	4.37
I feel that I am better prepared to implement differentiated instruction methods for my classroom.	3.89
My colleagues were respectful.	4.95
The instructors were informative and enthusiastic.	4.89
I would participate in another Coding for the Core project and recommend to others.	4.68
I feel that I am better prepared to teach CCSS-Mathematical Content standards.	4.06
I feel that I am better prepared to teach CCSS-Mathematical Practice Standards.	4
I feel that I am better prepared to assess my students' mathematics skills.	4
I feel that I am better prepared to select challenging mathematical tasks.	4.38
I feel that I am better prepared to use hands-on manipulatives to teach conceptual understanding of mathematical concepts.	4.38

All responses to the Likert-scale items were highly favorable. After participation in grant activities, all participants felt better prepared to teach using TEAM pedagogy (such as using critical thinking and problem-solving activities, assessing mathematics and science skills, and using differentiated instruction methods) and better prepared to teach using CCSS-Mathematics and Standards for Mathematical Practice. Many of the teachers had not been introduced to these standards nor did they use mathematics manipulatives for instruction prior to the training. All participants indicated that they would participate in another Coding for the Core project and recommend the program to others.

The first open-ended prompt was, "What are the top three takeaways you gained from your experiences with this project?" We isolated five themes from the 19 responses received, including opportunities for collaboration with other teachers, effective use of mathematics manipulatives, using problem-based tasks in the mathematics content sessions, integrating Lego robotics with mathematics curriculum, and formative assessment strategies. Table 5 includes representative quotes from teachers for each isolated theme.

Table 5
Top Takeaways from Coding for the Core

Theme	Representative Quotes
Opportunities for collaboration with other teachers. *n* = 11	• "The collaboration with colleagues from varied school systems has also been invaluable. While discussing strategies within your own school or system is important, you often find that your approaches are similar in nature due to the goals / push from your specific district. However, when you are exposed to teachers from other areas, you seem to get more diverse approaches to the same topics. I've also been given a lot of great resources to help me this year from these same colleagues." 7th-grade mathematics, 6 years, male • "…just being able to collaborate with other teachers in the same subject area and grade level as I am was a definite plus. Learning what works for other teachers and their students helps me to better help my students." 8th grade mathematics, 10 years, female
Effective use of mathematics manipulatives. *n* = 8	• "The biggest take-away for me was learning how best to use different manipulatives. I think that using manipulatives to teach a concept gives students the concrete foundation upon which they can build their knowledge and transition to an understanding of the abstract." 8th-grade mathematics, 10 years, female • "…the value of using manipulatives to aid in critical understanding as opposed to rule memorizing." 8th-grade mathematics, 1 year, female
Mathematics content sessions with the use of problem-based tasks. *n* = 8	• "All of the math 'tasks' that we did together during his sessions I felt like I could use in my middle school classroom. Also, it really helped to work out all of the problems, this way I am better able to help the students if they have questions." 7th-grade mathematics, 8 years, female • "Working to help students know why (developing conceptual knowledge) in math versus procedural knowledge is vital to long

Theme	Representative Quotes
	term learning and growth." 6th-grade mathematics, 9 years, female
Integrating Lego Robotics with curriculum. *n* = 7	• "I feel like the coding and working with the mind storms helped me to think more deeply about the problem-solving process which is helping me to incorporate projects and lessons that require multiple steps and more problem solving."6th-grade mathematics, 9 years, female • "Students can apply mathematical concepts through their learning of coding skills and robotics." 6th-grade mathematics, 5 years, male • "How to implement technology such as Lego Mindstorm kits in a useful, engaging way that is directly tied to our content."7th-grade mathematics, 6 years, male • "I am now very familiar with coding and LEGO robotics, enough to start a First Lego League competition team at my school. And to 'take away' a robot kit for my school was a major 'plus'!" 6th-grade science, 9 years, female • "This project has really opened up the idea of how to teach in math & science at my school. There are now a few other teachers in these subjects who have adopted the use of the Sphero. I honestly believe if it was not for this program, other people at my school wouldn't be as receptive in trying new things."7th-grade mathematics, 3 years, male
Formative Assessment Strategies *n* = 4	• "...how to incorporate math teaching methods and formative assessment better in my classroom."7th-grade mathematics, 5 years, female • "I learned some new, effective formative assessments and strategies that work with my teaching style and subject matter that I now use in my classroom. AND I have books with hundreds more ideas I can try myself!"6th-grade science, 9 years, female

Note. Each quote includes grade level, subject, years of experience teaching, and gender.

The second open-ended prompt was, "What parts of the training were the most helpful to you and why?" We isolated three areas of the training that were most helpful, including mathematics content sessions, Lego Mindstorms training, and mathematics pedagogy sessions. Table 6 includes representative quotes from teachers for each isolated theme.

Table 6
Most Beneficial Aspects of Coding for the Core Training

Theme	Representative Quotes
Mathematics Content Sessions n = 10	• "I think it is always great to learn new ways to teach things and new ways to explore. Also, by putting yourself in the students shoes you are able to see what types of things they may end up struggling with and prevent it before it happens." 7th-grade mathematics, 8 years, female • "The content lessons were amazing and informative. They challenged us and our thinking, and he was a great model for how we should teach and connect with our students in our classrooms. I learned many uses for Cuisenaire rods, algeblocks, barbies, graphing, geo boards, etc." 7th-grade mathematics, 5 years, female • "I loved the math content sessions for the same reasons, I liked seeing the different ways people solved and struggled with problems, it helped me to view my students in similar ways." 6th-grade mathematics, 9 years, female
Lego Mindstorms Training n = 8	• "I played with Lego's as a child, have a personal interest in engineering related topics, and a bachelor's degree in programming. The Mindstorm activities pulled all of that together for me." 7th-grade mathematics, 6 years, male • "Coding with Mindstorms programming because it has been the one I use on a daily basis. We have been able to get a total of 7 EV's in my room and my math enrichment class uses them on a daily basis." 7th-grade mathematics, 3 years, male • "Coding with Mindstorms and programming with bootstrap were very interesting and challenged me, that challenge is what kept me interested and has inspired me to find more ways to keep my teaching, lessons and classroom activities more engaging for my students." 6th-grade mathematics, 9 years, female

Theme	Representative Quotes
	• "I am using Mindstorms and LEGO robots in my class and would not have even considered that as an option had I not attended this training." Grades 6-8 STEM, 3 years, female • "Coding with Mindstorms: If I hadn't participated in this training, I would never have had the confidence or ability to be able to start the robotics team/club that we now have at our school. We have received a lot of positive feedback from the students and parents about this." 6th-grade science, 9 years, female
Mathematics Pedagogy Sessions *n* = 6	• "The pedagogy sessions have helped to prepare me to become a better teacher by sharing ways using technology in the classroom." 6th-grade mathematics, 6 years, female • "I also enjoyed the facilitated dialogue with other teachers from other schools, counties, and districts. It is always a blessing to learn from other people. I really liked the fact that we were a middle school group, and that we were placed in grade level PLCs for pedagogical discussion and lesson planning. My favorite activity was the chart paper, and discussing how we met indicators from the TEAM rubric. I definitely came away with a better use of how to incorporate technology into my classroom with ipads, plickers, kahoot, vimeo, and other suggestions teachers from other counties made." 7th-grade mathematics, 5 years, female
Note. Each quote includes grade level, subject, years of experience teaching, and gender.	

Successful aspects of Coding for the Core from analysis of the final project evaluation form included the structure of the workshops emphasizing instruction in both mathematics pedagogy and mathematics content with the effective use of mathematics manipulatives, formative assessment strategies, and problem-based mathematics tasks; creative integration of robotics and programming to teach common core mathematics content; and opportunities for teachers to collaborate.

Coding for the Core Challenges

The third open-ended prompt was, "What could we have done to improve your learning experience?" We isolated two areas of the training for which teachers suggested improvements, including changing the format of the Bootstrap Algebra training and requests for more direct time to program the Lego Mindstorms. Table 7 includes representative quotes from teachers for each isolated theme.

Table 7
Suggested Areas of Improvement for Coding for the Core Training

Theme	Representative Quotes
Programming Bootstrap was too fast and/or challenging (suggested changes in the format) *n* = 9	• "The programming with Bootstrap needed to be more basic and slower." 8th-grade mathematics, 13 years, female • "Bootstrap was difficult for me at the speed it was taught. A slower pace or a basic level may be helpful. This was my first experience with it so that may have been part of the difficulty." 6th-grade mathematics, 6 years, female • "Taken out bootstrap sessions or provided an alternative." 8th-grade mathematics, 1 year, female • "More assistance for struggling robotics and bootstrap learners would probably have been beneficial." 7th-grade mathematics, 5 years, female
More direct time programming the Lego Mindstorms *n* = 3	• "We could have spent more time learning how to code with Mindstorms and developing lessons for large groups of students to learn from in the classroom." 6th-grade mathematics, 5 years, male • "The only thing I wish I would have had the opportunity to do was to become more of an expert at creating lessons using the mindstorm software but to be fair, I have been able to access outside materials that have helped me in this process." 7th-grade mathematics, 3 years, male • "I would have liked to focus more on legos and less on bootstrap. The Legos were exciting but we could have done so much more. Have students (us) meet challenges, new builds, be creative. We learned a lot of the technical aspects of programming them but didn't get to put much of it to direct use." 6th-grade mathematics, 5 years, male
Note. Each quote includes grade level, subject, years of experience teaching, and gender.	

Nine of the 19 teachers who completed the final grant evaluation survey expressed concerns with the Bootstrap Algebra programming training. They felt the programming portion of the training was too challenging and needed to be delivered more slowly. Some even suggested that they would have preferred for that portion to be eliminated.

Discussion

Results as measured by the TPACK survey show an increase in participants' perceived ability to integrate technology in mathematics curriculum. After a Bonferroni correction was imposed, no individual TPACK construct showed a statistically significant change; however, the more notable positive changes between pre- and postassessment occurred in the constructs for TK, TCK Math, TPK, and TPACK Math. Technology was integrated each day within the context of mathematics with the use of Lego Mindstorms, Bootstrap Algebra programming, and digital instructional tools for teaching mathematics, such as virtual manipulatives or classroom response systems (Plickers and Kahoot).

Less notable changes occurred in teachers' pre-post TPACK scores for CK Math, PK, and PCK Math. While participants made significant increases in their mathematics content knowledge as measured by the content knowledge test, they made no significant change in their self-efficacy in mathematics content knowledge as measured by TPACK. They had confidence in their ability to understand and teach mathematics prior to participation.

The Mathematics Teachers TPACK Developmental Model can be used to make several observations about the teachers' reflections and activities after participation in the grant (Niess, et al., 2009). Teachers at the "Recognizing or knowledge" stage commented that they believed Lego Mindstorms and Bootstrap Algebra could be aligned with mathematics curriculum; however, they had not necessarily integrated them into the teaching and learning of mathematics.

Teachers at the "Accepting or persuasion" stage were using the robots as part of robotics teams or using the Hour of Code during computer labs for activities that were peripheral to classroom instruction. A seventh-grade teacher was working within the "Adapting or decision" stage by integrating the use of Lego Mindstorms with his mathematics enrichment classes and by having students use technology to enhance or reinforce ideas they had previously learned. Other teachers working at the "Adapting or decision" stage described instances of using the robotics and programming for demonstrations and modeling (i.e., functions and simple machines) for students and had not transitioned to providing experiences for students to use the tools on their own.

The lack of class sets of robots was a limiting factor for these teachers. A grades 6-8 STEM teacher was working within the "Exploring or implementation" stage by integrating Bootstrap Algebra programming as a means to engage the students in high-level thinking by using computer programming as a learning tool for mathematics. Other teachers working at this stage had students using coding and programming or robotics to explore algebraic reasoning, develop problem solving skills, and explore rates.

The Bootstrap Algebra curriculum was designed to be used at the middle school level and was directly aligned to Common Core mathematics curriculum. The problem-solving aspects were valuable to the teachers and reflected the ideal of providing a challenging problem for students (teachers) to solve and encouraged the mathematical practice of perseverance. We also liked the use of the Bootstrap Algebra curriculum because teachers and students have free access without the need for additional equipment beyond computers, whereas we were unable to buy class sets of the robots for the teachers with the nature of this grant. We wanted to include both in the training to give teachers more options of integrating programming and coding in the classroom.

That being said, the majority of the teachers enjoyed the programming and work with the Lego Mindstorms robots, and three teachers stated that they wished they had more time to really understand how to write programs using these robots. Given teacher's limited time for professional development, a solution would be to focus training on either Bootstrap Algebra or Lego Mindstorms so that the teachers have more time to become comfortable using them in their classrooms. Sustained time and training with either of these technology tools could provide the stimulus for teachers to successfully instruct with the tool at the adapting, exploring, and advancing stages of the Mathematics Teacher TPACK Development Model (Niess et al., 2009).

Many of these teachers' schools are located in rural settings, and opportunities to collaborate with other mathematics teachers at the same grade level are often rare. The structure of the professional development to allow for intensive grade-level interaction addressed a need that they were not able to fulfill within their own schools. Furthermore, one concern we had was the remote location of the training; however, only two teachers noted that the distance was a concern, although we suspect more teachers had this issue.

Roughly one fourth of the participating teachers taught in schools within close proximity to our training and greatly appreciated the location. They stated that they rarely attend PD offered close to their school systems beyond what is offered at the school and system level.

Limitations

We found that the teachers were eager participants in state-funded Improving Teacher Quality grant projects; however, two constraints proved to be challenges in transferring these types of activities into the hands of their students:

1. Lack of funding to purchase classroom sets of robotics and other supplies for programming and
2. Lack of sustained, long-term professional development as recommended in best practices literature of effective professional development (see, for example, Loucks-Horsley et al., 1996).

Additional limitations included recruiting and locating participants leading to a small sample size and getting the participants to follow through with fall workshop attendance.

We originally budgeted for 30 teachers; however, teachers seemed hesitant to sign up for the grant, possibly because they did not believe that programming and robotics could be used to teach mathematics. The syllabus was shared and included the alignment of state standards to the Bootstrap Algebra and Lego Mindstorms training and also indicated that half of the day would be used directly to teach content and pedagogy. A number of teachers applied for participation and then stated they would not be able to join because they had additional demands upon their time for training from their schools (i.e., intensive in-service training throughout the summer); therefore, we opened the training to middle school science and STEM teachers.

A final challenge we encountered was a drop-off in attendance for the two follow-up sessions during the fall semester. Some teachers missed out on training, and some were not able to complete postsurveys during the last session. We attempted to get the teachers to take the surveys (they were all online) on their own; however, we did not get all 22 participants to respond. We had hoped to collect the data after they had a chance to integrate techniques in their own classrooms. One teacher responded to the final evaluation prompt, "What could have been improved…" by stating that using a professional leave day from school would have been preferred to meeting on Saturday. This strategy could be a possibility for future grant initiatives.

Implications for Teacher Education

The ISTE (2017) *Standards for Educators* encouraged teachers to "create learning opportunities that challenge students to use a design process and CT to innovate and solve problems." America's 5-year strategy for STEM education depends upon developing and teaching digital literacy skills to solve complex problems (National Science and Technology Council, 2018). Preparing teachers to effectively use digital technologies and CT concepts with a particular emphasis on coding and programming requires adaptations to the design of professional development for in-service teachers who may not have had exposure to these tools and ideas in their preparation programs. This challenge calls for learning and teaching to be situated within a technology framework. The TPACK framework supports this challenge of integrating technology and CS effectively into mathematics instruction.

While learning computer programming has value in isolation, the process also provides an opportunity to explore the application of a variety of mathematical concepts in an interactive and engaging manner. CCSS-Mathematics topics such as ratios and proportional relationships, linear equations, functions, statistics, and probability can find exciting applications in areas such as Bootstrap Algebra video game design and Lego Mindstorms robotics. Teachers gained experience in communicating mathematical ideas effectively and applying those ideas by using manipulatives, computational thinking skills, mathematical models, and technology to solve practical problems.

As a result of participation, teachers had the opportunity to increase their TPACK, which ultimately may increase student achievement. They will be able to use their increased TPACK for the remainder of their career and continue to build on the foundation laid in the project. Additionally, the participating teachers will be able to serve as resources on mathematics and science computational thinking practices within their school systems.

Conclusions

Our results suggests a number of recommendations for teacher education. First, contextualizing CT activities within mathematics directly addresses the issue of students' self-selection into or out of CS classes. Second, the collective focus of integrating technology, pedagogy, and content into the design of the professional development and assisting the teachers in explicitly making connections to the TEAM rubric helped them focus on their specific contexts and transformed their TPACK throughout the training. These middle school teachers valued the time emphasized in the mathematics content sessions on problem-based, conceptually focused ways with the use of manipulatives that are often relegated only to the elementary school mathematics classroom.

Third, the focus on developing computational thinking practices within the context of mathematics through both Bootstrap Algebra and Lego Mindstorms robotics lessons was strengthened through the content aligned in the mathematics content sessions and reflected upon through the mathematics pedagogy sessions. If providing professional development that will extend a relatively short timeframe, we recommend that professional development efforts be focused either on Bootstrap Algebra or Lego Mindstorms — not both — to allow for more focused instruction.

Last, we recommend never to underestimate the value that teachers gain from opportunities to collaborate and communicate about their professional development experiences. The structure of the professional learning communities throughout the institute was among the top takeaways and experiences the participants valued.

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References

AIMS Education Foundation. (2006). *Bungee dolls*. https://store.aimsedu.org/item/da7414/bungee-dolls/1.html

Barr, V., & Stephenson, C. (2011). Bringing computational thinking to K-12: What is involved and what is the role of the computer science education community? *Inroads*, *2*(1), 48-54.

Bootstrap:Algebra (n. d.). *The Bootstrap:Algebra curriculum*. http://www.bootstrapworld.org/materials/courses/algebra

Buchholtz, N. (2019). Planning and conducting mixed methods studies in mathematics educational research. In G. Kaiser & N. Presmeg (Eds.), *Compendium for early career researchers in mathematics education* (pp. 131-152). Springer.

Carnegie Mellon University. (2019). *STEM network CS2N*. https://www.cs2n.org/curriculum

Computer Science Teachers Association. (2016). *K–12 computer science framework*. http://www.k12cs.org

Creswell, J. W., & Clark, V. L. P. (2017). *Designing and conducting mixed methods research*. Sage.

DeMonte, J. (2013). *High-quality professional development for teachers: Supporting teacher training to improve student learning*. https://www.americanprogress.org/wp-content/uploads/2013/07/DeMonteLearning4Teachers-1.pdf

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher*, *38*(3), 181-199.

ETA hand2mind. (2012). *Hands on standards, common core math*.

Gadanidis, G., Cendros, R., Floyd, L., & Namukasa, I. (2017). Computational thinking mathematics teacher education. *Contemporary Issues in Technology and Teacher Education, 17*(4), 458-477. https://citejournal.org/volume-17/issue-4-17/mathematics/computational-thinking-in-mathematics-teacher-education

Goode, J., Margolis, J., & Chapman, G. (2014, March). Curriculum is not enough: The educational theory and research foundation of the exploring computer science professional development model. In *Proceedings of the 45th ACM Technical Symposium on Computer Science Education* (pp. 493-498). Association for Computing Machinery.

Grover, S., & Pea, R. (2013). Computational thinking in K–12 a review of the state of the field. *Educational Researcher*, *42*(1), 38–43.

Hughes, J. (2005). The role of teacher knowledge and learning experiences in forming technology-integrated pedagogy. *Journal of Technology and Teacher Education*, *13*(2), 277-302.

International Society for Technology in Education. (2016). *ISTE standards for students*. https://www.iste.org/standards/for-students

ISTE. (2017). *ISTE standards for educators*. https://www.iste.org/standards/for-educators

ISTE. (2018). *ISTE standards for educators: Computational thinking competencies*. https://www.iste.org/standards/computational-thinking

Kee, D. (2013). *Classroom activities for the busy teacher: EV3*. CreateSpace Independent Publishing Platform.

Keeley, P., & Tobey, C. R. (2011). *Mathematics formative assessment: 75 practical strategies for linking assessment, instruction, and learning*. Corwin.

Koba, S., Wojnowski, B., & Yager, R. E. (Eds.). (2013). *Exemplary science: Best practices in professional development*. NSTA Press.

Koehler, M. J., Mishra, P., & Cain, W. (2013). What is technological pedagogical content knowledge? *Journal of Education*, *193*(30), 13-19.

Lappan, G. (2002). *Data about us: Statistics*. Prentice Hall.

Lappan, G., Fey, J. T., Fitzgerald, W., Friel, S. N., & Phillips, E. D. (2006). *Bits and pieces II unit*. *Connected mathematics, 2*. Pearson.

Lappan, G., Smith, M., & Jones, E. (Eds.). (2012). *Rich and engaging mathematical tasks: Grades 5-9*. National Council of Teachers of Mathematics.

LEGO Education. (2019). LEGO MINDSTORMS education EV3: Secondary. LEGO Education. https://education.lego.com/en-us/middle-school/intro/mindstorms-ev3

Lemov, D. (2015). *Teach like a champion 2.0: 62 techniques that put students on the path to college*. John Wiley & Sons.

Loucks-Horsley, S., Stiles, K., & Hewson, P. (1996). Principles of effective professional development for mathematics and science education: A synthesis of standards. *NISE Brief, 1*(1), n1.

Matherson, L. H., Wilson, E. K., & Wright, V. H. (2014). Need TPACK? Embrace sustained professional development. *Delta Kappa Gamma Bulletin*, *81*(1), 45-52.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, *108*(6), 1017-1054.

Muschla, J. A., Muschla, G. R., & Muschla, E. (2012). *Teaching the Common Core math standards with hands-on activities, grades 6-8*. John Wiley & Sons.

National Science and Technology Council. (2018). *Charting a course for success: America's strategy for STEM education*. https://www.whitehouse.gov/wp-content/uploads/2018/12/STEM-Education-Strategic-Plan-2018.pdf

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. http://www.corestandards.org/Math/

National Research Council. (2012). *A framework for K-12 science education: Practices, crosscutting concepts, and core ideas*. The National Academies Press.

NGSS Lead States. (2013). *Next generation science standards: For States, By States*. https://www.nextgenscience.org/

Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper, S. R., Johnston, C., Browning, C., Özgün-Koca, S. A., & Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education*, *9*(1), 4-24. https://citejournal.org/volume-9/issue-1-09/mathematics/mathematics-teacher-tpack-standards-and-development-model/

Ndongfack, M. N. (2015). TPACK constructs: A sustainable pathway for teachers professional development on technology adoption. *Creative Education*, *6*(16), 1697.

Olofson, M. W., Swallow, M. J., & Neumann, M. D. (2016). TPACKing: A constructivist framing of TPACK to analyze teachers' construction of knowledge. *Computers & Education*, *95*, 188–201.

Patton, M. Q. (1990). *Qualitative evaluation and research methods*. SAGE Publications, Inc.

Qin, H. (2009, March). Teaching computational thinking through bioinformatics to biology students. *ACM SIGCSE Bulletin, 41*(1), 188-191.

Richardson, S. (2009). Mathematics teachers' development, exploration, and advancement of technological pedagogical content knowledge in the teaching and learning of algebra. *Contemporary Issues in Technology and Teacher Education, 9*(2), 117–130. https://citejournal.org/volume-9/issue-2-09/mathematics/mathematics-teachers-development-exploration-and-advancement-of-technological-pedagogical-content-knowledge-in-the-teaching-and-learning-of-algebra

Schanzer, E., Fisler, K., & Krishnamurthi, S. (2013). Bootstrap: Going beyond programming in after-school computer science. *SPLASH Education Symposium*. https://web.cs.wpi.edu/~kfisler/Pubs/Bootstrap-SPLASHE-2013.pdf

Schanzer, E., Fisler, K., & Krishnamurthi, S. (2018, February). Assessing Bootstrap: Algebra students on scaffolded and unscaffolded word problems. In *Proceedings of the 49th ACM Technical Symposium on Computer Science Education* (pp. 8-13). Association for Computing Machinery.

Schanzer, E., Fisler, K., Krishnamurthi, S., & Felleisen, M. (2015, February). Transferring skills at solving word problems from computing to algebra through Bootstrap. In *Proceedings of the 46th ACM Technical Symposium on Computer Science Education* (pp. 616-621). Association for Computing Machinery.

Schmidt, D. A., Baran, E., Thompson, A. D., Koehler, M. J., Mishra, P., & Shin, T. (2009). Survey of preservice teachers' knowledge of teaching and technology. *Récupéré le*, *2*.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.

Tennessee Department of Education. (n.d.). TEAM: Tennessee educator acceleration model. https://team-tn.org/

Valk, L. (2014*). Lego Mindstorms Ev3 discovery book: A beginner's guide to building and programming robots*. No Starch Press.

Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K., Trouille, L., & Wilensky, U. (2016). Defining computational thinking for mathematics and science classrooms. *Journal of Science Education and Technology, 25*(1), 127-147.

Westphal, L. E. (2013). Differentiating instruction with menus for the inclusive classroom. Prufrock Press Inc.

Wing, J. (2011). Research notebook: Computational thinking—What and why. *The Link Magazine*, 20-23. https://www.cs.cmu.edu/link/research-notebook-computational-thinking-what-and-why

Yaşar, O. (2013). Teaching science through computation. *Generations*, *13*, 15.