# Helping Mathematics Teachers Develop Noticing Skills: Utilizing Smartphone Technology for One-on-One Teacher/Student Interviews 

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#### Abstract

Teaching mathematics for understanding requires listening to each student's mathematical thinking, best elicited in a one-on-one interview. Interviews are difficult to enact in a teacher's busy schedule, however. In this study, the authors utilize smartphone technology to help mathematics teachers interview a student in a virtual one-on-one setting. Free from physical constraints and preconceived biases, teachers can concentrate on building questioning, listening, and responding skills when noticing student mathematical thinking. Teachers engaged in four communication types when working with students through this technology: clarification, verification, and either extension or redirection.


Modern day mathematics teaching focuses heavily on inquiry. This sort of mathematics teaching, often labeled as reform-oriented or inquiry-oriented mathematics teaching, emphasizes conceptual understanding and procedural fluency as opposed to speed and recall (National Council of Teachers of Mathematics (2000). Teachers' instruction, therefore, revolves around understanding how students think, specifically the strategies that students create when trying to solve problems for the first time.

For instance, suppose a teacher is interested in how students solve a proportional reasoning problem, such as follows: "Teddy read 30 pages of a book in 45 minutes. How many pages should he be able to read in 120 minutes?" A general teaching flow for this type of problem might involve a proportion with an unknown variable, $30 / 45=\mathrm{x} / 120$, and then teaching cross-multiplication to form the equation $45 \mathrm{x}=(30)(120)$.

This approach, while efficient, limits students' understanding of why the teacher converted the proportion to an equation or how the procedure connects to the number of pages Teddy has read. Inquiry-oriented teaching, on the other hand, requires students to explore this problem on their own, to attempt to understand what is being asked, and to formulate a strategy. Then, the teacher can connect each student's strategies to other strategies in the class and, perhaps, to a general strategy.

Understanding exactly how a student solved a problem, unraveling the layers of steps and missteps a student took, however, requires a patience and attention aimed at individual students. This understanding does not happen when a student writes and explains a quick explanation on the board. Nor does it happen when a teacher circulates around the room, hovering over students as they work. Rather, the most effective way to understand and listen to the way a student thinks mathematically is through a one-on-one investigative interview between teacher and student, a technique honed by Piaget and referred to as aclinical interview (Ginsburg, 1997) or diagnostic interview (Huff \& Goodman, 2007).

Teachers rarely engage in these one-on-one interviews for a number of reasons (Zazkis \& Hazzan, 1998). First, teachers seldom have time in a busy school day to sit with a student for a one-on-one interview (Hunting, 1997). Second, learning how to question, listen, and respond to a student are highly refined teaching skills that do not simply manifest without organized support (J acobs, Lamb, \& Philipp, 2010). Yet, few teachers have access to support that helps them focus on noticing mathematical thinking. Third, whenever teachers work with students, certain student attributes affect their disposition toward that student (Dunn, 2004). That is, teachers cannot help but notice certain student characteristics, such as gender, ethnicity, familiarity with mathematical vocabulary, or even the clothes a student is wearing. These factors consciously and subconsciously affect how a teacher hears what a student is saying, inevitably creating prejudices that reify a teacher's perception of a student and obstruct an opportunity to focus on active listening of a student's mathematical thinking. Additionally, providing spaces for teachers to practice listening to children's mathematical thinking, particularly children they might not know or work with regularly, focuses teachers' attention completely on the child's thinking rather than subconsciously evaluating a student's physical attributes.

We attempt to address these problems by introducing an idea formulated in a current technology tool that brings the teacher-to-student interview into the modern era and helps to develop a teacher's mathematical noticing skills. We have built technology that allows teachers and students to interact without having to be physically next to each other, helping to mitigate pre-conceived biases so teachers can focus on building their skill in noticing student thinking.

In our study, we asked the research question: When using smartphone technology for a one on-one teacher/student mathematics interviews, what is revealed about how mathematics teachers notice through the way they question, listen to, and respond to student mathematical thinking?

## Literature Review

Recent research on teaching mathematics (J acobs et al., 2010; Smith \& Stein, 2011) as well as the Common Core State Standards for Mathematics Teaching Practices (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010) have outlined the importance of a particular set of teacher skills: the ability to question, listen to, and respond to a students' mathematical thinking. We refer to the termnoticing, particularly, mathematics teacher noticing, to encompass these constructs. While the act of noticing often refers to the ways a teacher attends to, interprets, and responds to students' thinking within a classroom environment (J acobs et al., 2010), we apply the construct of noticing to a one-on-one interview environment.

## Questioning

For teachers, learning how to question is the first step in attempting to understand a student's mathematical thinking. Questioning not only evaluates a student's mathematical knowledge, but also helps teachers understand how a student thinks (Aizikovitsh-Udi \& Star, 2011). Furthermore, good teacher questions help students in their own thinking by guiding their attention, loosening up their thinking, or forcing them to articulate their ideas (Smith \& Stein, 2011).

Good questioning involves focusing on concepts rather than calculations and allows for wait time after a student's response (Herbel-Eisenmann \& Cirillo, 2009). Therefore, in order for a platform to allow for effective mathematics teacher questioning, it must (a) allow for multiple types of questions that focus on student thinking and concepts and (b) integrate wait time that occurs after a student's response.

## Listening

Teaching mathematics for understanding requires responsive listening-not only to what a student says or shows, but also to what a student is thinking. Listening is the heart of effective mathematics teaching practice-the bridge between a teacher's questioning and subsequent response (Duckworth, 2001; Empson \& J acobs, 2008). Sherin and van Es's (2009) work on teacher video clubs found that when teachers stepped outside the classroom and learned to interpret events as opposed to judging whether they were good or bad, they were able to stop critiquing the teaching and focus on understanding what students knew. These teachers honed their listening skills through continually rewatching and discussing video of their own student/teacher interactions.

Furthermore, Empson and J acobs (2008) found that learning to listen required repeated viewings of supported student/teacher one-on-one interactions. Therefore, any platform that allows for teacher listening/ noticing must (a) move outside the whole-group classroom environment, (b) help teachers learn to interpret their own interactions with students, as opposed to evaluate them, and (c) allow teachers multiple viewings of their interactions with students.

## Responding

Of the three noticing skills, learning to respond to children's thinking is the hardest skill for teachers to develop, yet the most effective in extending student learning (J acobs \& Ambrose, 2008). While teachers develop questioning and listening skills over time, the ability to respond does not automatically develop with experience. Smith and Stein (2011) found that teachers needed extensive sustained practice in learning how to respond to
student thinking in order to develop a repertoire of appropriate responses. Therefore, any platform that helps teachers respond to student mathematical thinking must provide ample opportunity and support for teachers to practice how to respond.

## Technology Framework

Our work extends prior scholarship on mathematics teacher noticing, particular in its subconstructs of how teachers question, listen, and respond to students within a one-onone interview setting. Little work exists exploring the nuances of the one-on-one mathematics interview, which may be because previous technologies did not allow for precision in capturing data within the interviewitself (Hunting, 1997).

The technological features that seem necessary in order to explore mathematics teacher noticing within a problem solving interview include (a) the capture of every utterance or artifact, (b) ability to re-watch anything previously written or recorded, (c) virtual interactions so both parties do not have to be physically present, and (d) immediate access of a fellow math educator for teacher support.

By using existing technology found in smartphones, we have developed a system that captures every artifact and utterance made between teacher and student. These small words, phrases, and diagrams are crucial in understanding how teachers and students talk to each other. Previously, capturing every audio utterance or facial expression required massive amounts of video and audio data collection and storage. Through utilizing smart phones as our communication conduit, all utterances are documented as part of the back-and-forth nature of mobile chats, revealing a digital transcript of text messages, images, and video that teachers and students use to discuss and share their mathematical thinking.

Second, smartphone textual, photo, and video-based communications create a visual record that both teacher and student can recall and refer to in the midst of their interview. This ability to watch a student's explanation repeatedly or to scroll back in time adds precision to the interview.

Third, our technology situates the interview virtually, so the teacher and student never see each other. By physically separating teacher and student, we attempt to mitigate preconceived notions that both teacher and student might make about each other in a teacher training exercise to develop their noticing skills.

Finally, a fellow mathematics educator is available for immediate access to the teacher for support, either physically or through the technology itself. The teacher can ask for guidance as to how to question, listen to, or respond to student thinking, or to articulate next steps.

We find the use of smartphone-based technology beneficial when working with current pre- and in-service mathematics teachers for three reasons.

- Using a smartphone to focus on mathematical student thinking is within the normal, everyday use of smartphones for today's teachers.
- Our technology works on currently available and accessible technology; teachers do not have to acquire new equipment, hardware, or software.
- The technology is simple: a conduit of video, images, and text untethered to a particular phone or operating system.


## Research Methods

The purpose of this exploratory study was to investigate how teachers notice students' mathematical thinking within a one-on-one interview when given the opportunity to use technology. We collected qualitative data to gain a better understand of how teachers question, listen to, and respond to student thinking, as well as their feedback to the entire virtual interaction.

## Participants

Our study involved three mathematics teachers in the northeastern region of the United States engaged in a one-on-one interview with a student using our technology. Teachers were recruited through existing networks of mathematics teachers that the authors were members of, being mathematics teacher educators and professional developers. Teachers were selected based on interest in piloting technology that helps teachers learn to listen to student mathematical thinking.Mildred had taught seventh and eighth-grade for less than 5 years at a public science, technology, engineering, and mathematics (STEM) focused magnet middle school. Justin had taught ninth-grade algebra and 12th-grade calculus and precalculus at a public high school for less than 5 years. Skyler had taught 2nd and 1st-grade in a private elementary school for 5 years.

## Data Collection

Teachers each used their smartphones to interact with what they believed to be a student who shared a video of how he solved a prechosen mathematical problem. These initial student videos were under 2 minutes each and consisted of point-of-view shots from a student's perspective of how he attempted to solve the mathematical task.

The student for this study was the first author pretending to be a student. This deception was necessary in order to create an idealized case to explore how teachers interacted with this technology. To do this, we met as a research team to detail specifically how the student should respond to provide systematic strategy, thinking, and responses. By controlling exactly what the student did and said, we were able to focus on the teacher and examine what might be possible with this technology. In the videos, one of the authors had his 5-year old daughter play the part of the elementary student. For ease of reading, we refer to the researcher deceptively interacting with the teacher as "the student."

Before the student interviewbegan, the teachers were given the opportunity to review and attempt to solve the task and ask questions about the process. Then, each teacher sat with a member of the research team for a 30-60 minute student interview session, interacting with the student through a smartphone. During the student interview session, we videotaped the teachers in order to capture their reactions and responses to the student. We refer to these data as the "teacher video," whereas all other references to video refer to those sent between the teacher and student about the mathematics problem.

The student interview session began when the student sent the teacher a video of how he or she solved or attempted to solve a mathematical task. The videos involved the student narrating his or her strategy for the problem, pointing to written work on paper. Each video was shot from the student's perspective, with the student's finger pointing out specific parts of his or her strategy during the course of the explanation.

For Mildred and Justin, who work with secondary students, the student worked with a ratio and proportional reasoning problem: "Teddy read 30 pages of a book in 45 minutes. How many pages should he be able to read in 120 minutes?" (See Video 1.)


Video 1. A "student" sent to secondary teachers Mildred and Justin to solve the task, "Teddy read 30 pages of a book in 45 minutes. How many pages should he be able to read in 120 minutes?" The student has already written out his strategy and uses his finger and voice to explain his thinking, mimicking a high school student's confusing explanation. This video can also be viewed at https:/ / vid.me/ GQ3R.

For Skyler, who works with young elementary students, the student worked with the equal sharing problem: "Three children share 10 mini-burritos so that each child gets the same amount. How much burrito does each child get?" Skylar was told that her student was sitting with an adult researcher, who helped her make the videos and type the text messages back to Skylar. (See Video 2.)

We chose the tasks because the content would be familiar to teachers and the student reasoning would be potentially complex and in need of unpacking, based upon the literature of how students approach these problems (Ben-Chaim, Fey, Fitzgerald, Benedetto, \& Miller, 1998; Empson, 1999; Empson \& Levi, 2011; Tourniaire \& Pulos, 1985). These tasks were also chosen because they had an initially high level of cognitive demand (Smith \& Stein, 2011), were applicable to student strategy comparison (RittleJohnson \& Star, 2007), and contained the possibility for teachers to notice mathematical thinking (J acobs et al., 2010).


Video 2. Initial video that a "student" sent to elementary teacher Skyler to solve the task, "Three children share 10 mini-burritos so that each child gets the same amount. How much burrito does each child get?" The student has drawn three children, each with a number of small ellipses (burritos) in each child's belly. The student has written out, "Every child gets 3 . Cut last burrito in 3. Understand?" The student here is played by one author's 5 -year old daughter. This video can also be viewed at https:// vid.me/ hcp0.

Throughout the interview, we asked the teachers to think aloud in their interaction with the student about what they noticed and what question they might ask the student. The teachers were then told that before they gave the student feedback, they had the opportunity to ask the student some clarifying questions. The purpose for this strategy was to help the teachers take time to reflect on the student's mathematical thinking and to make sure they could engage the student in a meaningful discussion about his thinking.

This phase of the session involved the teacher and student text messaging back and forth to clarify specifics about the student's strategy. If the teacher did not want to ask questions, the researcher asked the teacher to explain what the teacher understood about the student's strategy.

Next, when the teachers felt they understood the student's strategy, the research team member asked them what their plan was for the feedback and to articulate a goal or goals for the communication. The researcher asked questions such as, "In an idealized environment free from the extra pressures and noise of the classroom, what would you want the student to explain? What would you want the student to know or be able to do?"

For example, for the proportional reasoning problem, Mildred wanted the student to find the solution using cross multiplication. The researcher informed Mildred that the discussion would stop when she felt the student was where she wanted with using cross multiplication as a strategy. Teachers then asked follow-up questions, which could involve unpacking the student's strategy, asking students to make additional videos or solving the tasks using a different strategy.

We aligned our guidance with our noticing framework: teachers asked questions to attend to and interpret a student's thinking, then articulated their feedback plan on how they would respond to the student. We also provided time for each teacher to reflect on the interaction in order to make sense of the mathematical thinking exhibited. This step involved the teacher and student sending texts, videos, and images back and forth to clarify specifics about the student's strategy, the teacher asking students to solve the same tasks using a different strategy, and the teacher asking the student to make additional videos (see Figure 1).

After the interaction, we asked the teachers postinterview questions about the interaction, such as "How did you feel about this medium in terms of engaging with the student?" or "In what ways did the video help you understand that is different than what you might notice in the classroom?" Mildred said she would be scared to use this medium with her students, revealing that she is extremely cautious in how she communicates with students outside of the classroom. J ustin liked the interactions, finding that it forced him to think more about what his student said. Skylar liked focusing her attention on only one student's thinking, noting that she could understand the student well because her voice was clear.

|  | I see where you doubled <br> the pages and minutes to <br> find larger chunks but <br> where did you get 10p/ <br> 15m? |
| :--- | :--- |
| Teacher | So what led you to <br> subtract those numbers? <br> What arithmetic did you <br> use to go from your base <br> rate to 15 minutes and 10 <br> pages? |
| What allowed you to divide <br> by any number? I thought <br> it was just 30 minutes for <br> 45 pages. And why didn't <br> dividing by two "work"? |  |



> I knew that 90 pages in
> 135 minutes was too big.
> So I needed to subtract 15
> minutes from one side,
> and 10 pages from the other side.

Stu

> I knew 30 pages was 45
> min. So first I divides both sides by 2 . That didn't help. Then I divided by 3 .

Figure 1. A teacher and student engaged in smartphone-based problem solving interview, in which the teacher watches a video of the student's strategy and then questions the student about it. Notice that the teacher is asking questions to clarify and verify his interpretation of the student's strategy.

## Analysis

We used thematic analysis to analyze the data collected during the interviews. Thematic analysis is a "common general approach to analyzing qualitative data that does not rely on the specialized procedures of other means of analysis" (Schwandt, 2007, p. 291). By using this approach and grounding our thinking in teachers' professional noticing literature, we were able to identify and code the different communication types used by the participants.

Our data consisted of the teacher video; the text, photo, and video messages sent back and forth between the teacher and student; and any written artifacts from the interaction. We started with a theoretical framework that predicted the teachers would engage in either clarification or verification questions (defined in Table 1). We used this skeletal framework to analyze the interaction of each teacher.

To analyze the data, two members of the research team watched each teacher video together. In these meetings, we frequently stopped the videos, wrote analytical memos, and engaged in discussions about what we noticed. For critical moments during the interview, we generated transcripts detailing the rhythm of each interaction as well as what we saw in terms of what the teachers noticed about student mathematical thinking. We paid attention to each teacher's facial expressions, vocal tone, and the speed with which they responded to students.

Essentially, we micro-analyzed the specific visual and textual artifacts and utterances shared between teacher and students through our technology, as well as what the teacher was doing during these moments of sharing (as in Corbin \& Strauss, 2008). When we were finished with this analysis, we went back to all the analytical memos with the purpose of understanding if and how our ideas regarding teacher-student interaction were related to teacher noticing. After discussion between research team members, we decided that we should add two additional categories to our framework: extension and redirection (defined in Table 1).

As a result of this analysis, we settled on four communication types related to mathematics teacher noticing. We then went back into each teacher's interaction to create a map of how the teachers navigated between these four communication types. These communication types might not be generalizable to all teachers because it is based only on our analysis of three teachers (Erlandson, Harris, Skipper, \& Allen, 1993). Through the use of thick description and micro-analysis of the interactions, however, we can confidently create a theoretical framework for further inquiry into this new space of using technology for one-on-one teacher-student mathematics interviews.

## Results

We started this work to explore how teachers could use technology within a one-on-one interview and what this technology could unveil so we could better understand the way teachers and students communicate about mathematical thinking. The three teachers each showed different ways of engaging in mathematical noticing when using this interview technology. The middle school teacher, Mildred, kept trying to push the student toward a traditional, algebraic algorithm applicable to all cases. The high school teacher, Justin, expressed interest in using this technology in an ideal classroom so he could spend more time probing and extending individual student problem solving. The elementary teacher, Skyler, straddled the traditional versus inquiry-oriented fence by
trying to listen to the student's unique strategy but was hesitant to accept it because it was neither efficient nor applicable to all cases.

We noticed that teachers combined listening with questioning and responding, meaning that the way they questioned or responded was based on the way they listened. Therefore, in connecting to the literature around mathematics teacher noticing, we categorized two communication types of teacher questioning, clarification and verification, and two communication types of teacher listening, extension and redirection. (See Table 1 for definitions and examples of each type of communication.)

| $\begin{array}{l}\text { Type of } \\ \text { Communication }\end{array}$ | $\begin{array}{l}\text { Definition }\end{array}$ | Example |
| :--- | :--- | :--- | \left\lvert\, \(\left.\begin{array}{ll}Clarification \& \begin{array}{l}When the teacher tries to <br>

understand exactly what a student <br>
did and why, in order to fill in <br>
missing details or clarify <br>
ambiguous video, audio, or text <br>
from a student.\end{array}\end{array} $$
\begin{array}{l}\text { Okay, can you clarify how you } \\
\text { decided to subtract 10 pages and } \\
\text { 15 minutes? - Mildred Tell me } \\
\text { how you decided that each friend } \\
\text { started with 3 burritos. - Skylar }\end{array}
$$\right.\right\}\)

When teachers questioned the student through this interface, they were either clarifying or verifying. Clarification involved trying to understand exactly what a student did and why in order to fill in missing details or clarify ambiguous video, audio, or text from a student. Verification involved the teacher asking confirmation questions about the student's strategy. Verification is similar to the revoice classroom talk move, in which a
teacher restates a student's contribution for the rest of the classroom to hear and then asks the student to verify if this is what he or she meant (Chapin, O'Connor, \& Anderson, 2003). Verification also requires teachers to pay attention to the detail in student thinking.

For example, J ustin used verification questions to confirm his student's strategy, texting, "So you can manipulate the rate by any amount, and you didn't like 22.5 because it didn't divide evenly into 120 . Is that still the number of pages he can read in 15 minutes?" J ustin asked this verification not only to confirm his own understanding, but also to help his student verify this thinking to himself. Therefore, verification served dual purposes: helping the teacher understanding a student strategy as well as helping a student understand his or her own strategy.

Justin's question eventually led to a text sequence in which the student came to a more general strategy, which was the teacher's goal throughout this interaction. The teachers found this technology allowed them to focus on clarification and verification, since they could watch the videos multiple times and review every piece of communication in order to hone in on opaque explanations, asking students to make additional videos or diagrams that better explained their thinking. Verification questions also showed that teachers were actively listening and attending to students' strategies.

After questioning, teachers responded to students using either extension or redirection. Extension broadened the original problem in order to build upon the student's strategy. For example, Justin tried to extend the problem using numbers in which the student's original strategy would break. He texted, "So what if I read at the same rate as Teddy and read for 100 minutes? How many pages? And what if I read for 84 minutes?"

This type of communication required the ability to make inferences about what a student understood and then formulate questions that probed into this thinking. Extension communication moved beyond the initial problem, pushing the student to look at new situations in order to generalize or alter the original strategy or see new aspects of this strategy. Justin told us that this technology allowed him to push his student further than if he was in the classroom, because he was able to review the student video and continually ask questions to the student without feeling like he was interrupting.

Justin also felt able to present extension problems that naturally flowed within the course of the interaction. Of the three teachers, only Justin engaged in this type of communication, confirming how difficult and rare this type of noticing is among teachers (Empson \& J acobs, 2008; J acobs et al., 2010; Sherin \& van Es, 2009). Furthermore, Justin kept looking to us for guidance in asking these sorts of questions to his student. Even with technology that facilitates this communication, he said that he needed another educator present to talk with to articulate these extension questions.

Teachers also responded with redirection, cutting off a student's explanation in order to redirect a student toward a teacher-presented strategy. We had not anticipated this type of question, which was used by Mildred and Skylar. For example, Mildred texted, "Ok, do you think you could set this problem up a different way? How can you solve it algebraically?" Additionally, Skylar redirected her student to use fractional language and notation rather than continue with an equal partitioning strategy, texting "Can you write as a fraction how much each girl got to eat?" Redirection occurs when teachers feel they have heard enough of what a student is thinking or think the student is headed down an incorrect path and decide to intervene by telling the student to use a particular strategy.

Extension and redirection are mutually exclusive. A teacher can either extend a student's mathematical thinking or redirect a student toward a particular strategy. What influences a teachers' choices to extend or redirect stemmed from whether they were, in fact, actively listening, which could be assessed in their perception of the utility of this activity.

For example, Justin viewed the interview as an opportunity to explore and probe a student's thinking and to listen. He thought the implementation of the interview was powerful because it opened up an opportunity to engage with the student, to really look at every artifact of student thinking and discuss mathematics with the student. As a result, he engaged in extension communication to help the student augment the original strategy into a more general one.

Mildred, on the other hand, saw the interview as an intervention or tutoring tool to help struggling students catch up to the rest of the class. Similarly, Skylar saw the interview as an assessment tool, comparing it to miscue analysis that occurs in reading instruction. (Miscue Analysis is a diagnostic tool used to assess a students' reading process. Miscues are not mistakes, but opportunities to gain insight into student's linguistic strengths and meaning making, Goodman, 1969). As a result, both Mildred and Skyler used the interview to redirect students towards strategies unrelated to the students' original strategy.

Another interesting result involved the embedded wait time. The process of students and teachers sending each other videos and text messages is much slower than a typical face-to-face interview. Much of the interview was spent waiting for the other person to respond. Justin found this to be extremely helpful in the interaction, as it allowed both student and teacher to take the time to articulate their thinking before writing it down or making a video. While this process is slower than a standard face-to-face interview, it directly embedded the good questioning technique of wait time after a student responds that Herbel-Eisenmann and Cirillo (2009) pointed out as necessary.

We answered our original research question by finding that in a technology-enabled one-on-one interview mathematics teachers interacting with students might use two types of questioning strategies (clarification and verification), and two types of responding strategies (extension and redirection). The technology zoomed into the specific utterances and in-the-moment thinking surrounding the ways teachers interpret what a student is saying. This study adds to the growing body of literature examining exactly how mathematics teachers can support student mathematical thinking through the way they speak, respond to, and listen to students. Our technology is not used as a means to promote this type of discourse, but rather as an investigational tool to isolate and analyze an interaction that often happens too quickly to study and unpack in a traditional classroom setting.

## Conclusion

The way we used technology in this study offers an alternate glimpse into the teacher/ student interaction compared to a standard face-to-face interview. By allowing teachers and students to interact without having to be physically next to each other, we hoped to mitigate preconceived biases. Teachers initially had only the student video to examine and no other data about the student they worked with (e.g., noticing student's skin color, method of dress, gender, etc.). Teachers, therefore, had to focus only on the students' thinking, which is a different lens than they would use in a traditional classroom setting in which a teacher notices what a student looks like before getting to know their thinking (Dunn, 2005; White, Murray, \& Brunaud-Vega, 2012). None of our teachers mentioned or attended to the students' gender, race, ethnicity, or background in their
responses. Even when specifically asked about what they might know about the student in the postinterview, each teacher mentioned being frustrated because all they could see was the student's hand or hear the student's voice, unable to make many conclusions from this information. We are not claiming this technology mitigates teacher bias, but each teacher felt frustrated by the lack of physical attributes they could use to know the student, forced to focus only on the text, images, and video that the student shared to draw a conclusion about the student. Even being able to see the student's skin color through the student's hand on the video or being able to hear the student's voice did little to ease the frustration the three teachers felt.

The biggest constraint we found in our study was whether or not teachers would actually use this type of technology with their students. These interviews require much more time than a traditional face-to-face interview, so expecting teachers to engage in these extensive activities with each of their students might be unreasonable. Rather, perhaps the best place for this sort of technology is with teacher educators or professional developers working with preservice and in-service teachers, perhaps in a mathematics teaching methods or content course or in a professional development course focused on noticing student's mathematical thinking.

One affordance this technology demonstrated was a way for teachers to immerse themselves in student's mathematical thought beyond what they would see in the classroom. Our technology isolated a teacher-student interaction to its essence, and thereby served as a mechanism for a teacher to engage in active listening of student thinking and extend a student's mathematical thinking. In order to accomplish this, we will hereafter present this tool as an opportunity to probe and extend student thinking and not as an assessment tool.

In our analysis, we were correct in predicting that teachers would either respond to the student's strategy video with either a clarification and verification question. Both these types of communication were a teacher's way to probe deeper about the student's strategy. That the teachers used either one of these types of questions was not surprising and showed, on the surface, how teachers were attending to and interpreting a student's mathematical thinking (J acobs et al., 2010).

The ways teachers engaged in responding to student's mathematical thinking with either extension or redirection communication was surprising. Two of our three teachers decided to redirect, indicating that they might not be attending to and interpreting student thinking, but rather listening to see if the student's strategy matched their own. According to J acobs and colleagues, attending to children's strategies includes being able to recall the mathematical details of the strategies in order to discern children's understandings of a particular concept.

Interpreting student thinking includes having consistent reasoning about a student's understanding and a specific strategy that is connected to research on mathematical development. Therefore, by redirecting, the teachers may not have had well-developed noticing skills of attending and interpreting. Their communications with the student did not focus on the mathematical details of the student's strategy, was evaluative, and focused on general teaching and learning. Therefore, through this technology, we can see how some teachers might focus more on a redirection-focused response rather than on expanding the student's thinking.

Justin, on the other hand, showed a different response by attempting to extend the student's thinking. Earlier, we wondered if this difference was due to the ways each teacher viewed the purpose of this technology. Did they see it as a diagnostic tool for
helping struggling students or a communication tool for engaging students in a mathematical conversation? The latter is the far more useful purpose of this technology, a way for teachers and students to engage in mathematical dialog that extends the student's mathematical thinking in ways that might be difficult to do in the classroom.

As we continue our work using technology to assist in teacher noticing of student mathematical thinking, we must caution that using this type of technology to diagnose or assess student thinking robs both teacher and student of the opportunity to engage in open mathematical dialog, of the opportunity to actually do math together. Rather, technology for teacher noticing of student mathematical thinking must give students agency to articulate, revise, and extend their own thinking. Only then will the technology serve as a helpful tool for mathematics learning, rather than another means to ignore a student's voice.

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